Integrated Robust Identification and Control of Large-Scale Processes

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We propose the use of *pseudo-singular values*, which are closely related to singular values but are allowed to have sign, as a convenient approach for developing techniques for the identification and control of large-scale processes. Steady-state controllability can be assessed directly in terms of the pseudo-singular values. It is shown that to control an output disturbance direction with zero steady-state error it is necessary to correctly identify the sign of the corresponding pseudo-singular value. At the identification stage, this motivates the estimation of confidence intervals for the pseudo-singular values from input−output data. A controller with integral action should not attempt to manipulate in process input directions corresponding to output disturbance directions that cannot be controlled with confidence. These principles motivate a controller structure appropriate for providing the robust control of poorly conditioned large-scale processes. Any controller design technique can be applied to produce a controller with the proposed controller structure. The controllability results and the integrated identification/controller design procedure are illustrated using an industrial paper machine example.

1. Introduction

Needs for increased product quality, reduced pollution, and reduced energy and material consumption are driving enhanced process integration. This increases the number of manipulated, measured, and controlled variables that must be jointly handled by the control system. Using a combination of theoretical results and numerical experiments, Braatz (Braatz, 1997) showed that large-scale processes are almost always poorly conditioned; that is, the condition number of the plant transfer function matrix is large. It is well-known that poorly conditioned processes can be difficult to control. Note that the condition number *by itself* does not provide a meaningful measure of the difficulty in controlling a multivariable process, since the controllability of the process based on the model depends also on the quantity of input−output data and the variance of the stochastic disturbance/noise signals (other weaknesses in using a naive condition number analysis are described in the literature (Braatz and Lee, 1996; Skogestad et al., 1990).

Eigenvalues and singular values have been used to study the identification of poorly conditioned processes (Koung and MacGregor, 1993, 1994; Li and Lee, 1994). It was recently shown that *pseudo-singular values*, which are closely related to singular values but are allowed to have sign, provide a more convenient approach for developing techniques for the identification and control of large-scale sheet and film processes (Braatz and Featherstone, 1995; Featherstone and Braatz, 1995). Here we extend the definition of pseudo-singular values for application to general processes and show these tools are useful for both understanding and quantifying steady-state controllability. In terms of *understanding*, the pseudo-singular values provide a more natural multivariable generalization of the single-input single-output (SISO) concept of the requirement to “correctly identify the sign of the steady-state gain” than provided by singular values. In particular, it is shown that to control an output disturbance direction with zero steady-state error it is necessary to correctly identify the sign of the corresponding pseudo-singular value. In terms of *quantifying* steady-state controllability, an expression is derived for calculating confidence intervals for the estimated pseudo-singular values. This assessment of controllability is computed directly from the input−output data and is not an explicit function of the condition number.

The importance of correctly identifying the signs of the pseudo-singular values leads to the development of a novel additive uncertainty description, with the input weight being the nominal input rotation matrix, the output weight being the nominal output rotation matrix, and additional diagonal weights being defined by the confidence intervals for the pseudo-singular values. It is argued that this uncertainty description is not conservative—this is in sharp contrast to other multivariable uncertainty descriptions. Also, the weights for this uncertainty description are directly specified from the input−output data.

Any controller with integral action should not manipulate in process input directions corresponding to output disturbance directions that cannot be controlled with confidence—otherwise instability or poor performance will result. The extent that a controller can manipulate in an output disturbance direction whose sign of the corresponding pseudo-singular value is known with confidence is defined by the size of the confidence interval. These principles motivate a controller structure appropriate for providing robust control of poorly conditioned large-scale processes. Any multivariable controller design technique can be applied to produce a controller with the proposed controller structure. The controllability results and the integrated

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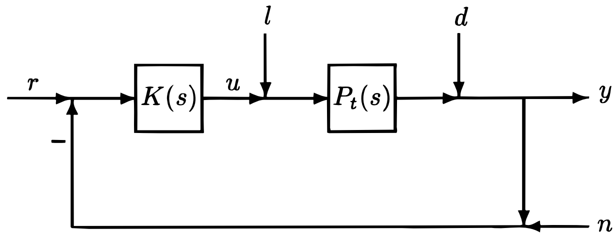

Figure 1. Standard feedback control system. The manipulated variable is u, the process output is y, the setpoint is r, the measurement noise is n, and the disturbances are d and l.

identification/controller design procedure are illustrated using an industrial paper machine example.

2. Pseudo-Singular Values

Consider a general $n \times n$ transfer function matrix $\mathbf{P}(s)$ that relates the manipulated variables u to the controlled variables y. Manipulated variables, controlled variables, and disturbances are assumed to be scaled as described in the literature (Skogestad and Postlethwaite, 1996). All of the poles of $\mathbf{P}(s)$ will be assumed to be in the open left half-plane; that is, $\mathbf{P}(s)$ is strictly stable. The matrix $\mathbf{P}(s)$ is assumed to be square for simplicity in notation only; all definitions and results in the manuscript are generalized to nonsquare $\mathbf{P}(s)$ by appending either extra rows or columns of zeros to the transfer function matrix. The real singular value decomposition of the steady-state gain matrix is defined by (Golub and van Loan, 1983)

$$\mathbf{P}(0) = \hat{\mathbf{U}}\mathbf{\Sigma}\hat{\mathbf{V}}^{\mathrm{T}} \tag{1}$$

where $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ are constant real orthogonal matrices, and $\mathbf{\Sigma}$ is a constant real diagonal matrix whose diagonal elements are referred to as *singular values*. The singular values are ordered and nonnegative; that is, $\Sigma_{11} \geq \Sigma_{22} \geq \ldots \geq \Sigma_{nn} \geq 0$. The singular values (and hence $\mathbf{\Sigma}$) are unique for a given matrix $\mathbf{P}(0)$, while the $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ matrices are nonunique. For example, multiplying the ith columns of $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ by -1 results in an additional pair of matrices ($\hat{\mathbf{U}}$, $\hat{\mathbf{V}}$) that satisfy (1). The singular value decomposition of the transfer function matrix, both at steady state and as a function of frequency, has been applied to chemical process control problems for more than 10 years (Downs and Moore, 1981; Kaspar and Ray 1993; Lau et al., 1985; Moore 1986; Ogunnaike and Ray, 1994; Skogestad et al., 1988).

A well-known controllability result for stable single-input single-output (SISO) processes is as follows (this is a variation on a result in the literature (Morari, 1985)).

Lemma 1. *Assume that the true plant ($\mathbf{P}_t$) and the model of the plant ($\mathbf{P}_m$) are strictly stable proper SISO transfer functions. Then there exists a controller with integral action that stabilizes both $\mathbf{P}_m(s)$ and $\mathbf{P}_t(s)$ if and only if $\mathbf{P}_t(0)/\mathbf{P}_m(0) > 0$.*

An *integral controller* will refer to a controller with integral action ($\mathbf{K}(s) = (1/s)C(s)$ in Figure 1, where $\det(C(0)) \neq 0$). Lemma 1 indicates that it is necessary for the sign of the steady-state gain to be correct for a model-based integral controller to stabilize a stable linear process. Garcia and Morari (1985a) provided a generalization of Lemma 1 appropriate for multivariable controllers that are required to maintain stability with detuning. The conditions were in terms of eigenvalues and were further studied by Koung and MacGregor (1993, 1994). The main weakness of the eigenvalue conditions is that they consisted of a coupling between the plant model and the true plant which is highly cumbersome for use in robust control analysis and design. An additional concern is that eigenvalue conditions can lead to misleading indications of stability robustness, as discussed in Doyle and Stein's classic paper (Doyle and Stein, 1981). While the singular value decomposition is a powerful tool in any control engineer's toolbox, there is no convenient way to use singular values to generalize Lemma 1 to multivariable processes since the singular values are *always* nonnegative. This motivates the definition of the pseudo-singular values.

Define the diagonal matrix $\mathbf{D}_U$ which has each diagonal element either $+1$ or -1, with the (i,i)th element being -1 if the dot product $(\hat{V}^i)^{\mathrm{T}}\hat{U}^i$ is negative (matrices $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ are defined in (1), and A^i refers to the ith column of the matrix $\mathbf{A}$). Then

$$\mathbf{P}(0) = \hat{\mathbf{U}}\mathbf{\Sigma}\hat{\mathbf{V}}^{\mathrm{T}} = (\hat{\mathbf{U}}\mathbf{D}_{\mathbf{U}})(\mathbf{D}_{\mathbf{U}}\mathbf{\Sigma})(\hat{\mathbf{V}})^{\mathrm{T}} = \mathbf{U}\mathbf{\Lambda}(0)\ \mathbf{V}^{\mathrm{T}} \tag{2}$$

where $\mathbf{\Lambda}(0) = \mathbf{D}_{\mathbf{U}}\mathbf{\Sigma}$ is a constant real diagonal matrix whose diagonal elements will be referred to as *pseudo-singular values*. The pseudo-singular values can be of any sign (including zero) and are defined such that the angle between the corresponding U^i and V^i is not greater than 90°. The right-hand side of (2) will be referred to as the *pseudo-singular value decomposition* (pseudo-SVD).

The unique matrix $\mathbf{\Lambda}(0)$ is the steady-state matrix for a transfer function matrix $\mathbf{\Lambda}(s)$ defined by

$$\mathbf{\Lambda}(s) = \mathbf{U}^{\mathrm{T}}\mathbf{P}(s)\ \mathbf{V} \tag{3}$$

which will be referred to as the *pseudo-singular value matrix*. While $\mathbf{\Lambda}(s)$ is diagonal at steady state, in general it is not diagonal for other values of s. Each off-diagonal element is a transfer function whose steady-state value is zero.

The term "pseudo-singular values" was first proposed by Featherstone and Braatz (1995) in reference to a class of industrially-relevant processes referred to as *pseudo-SVD processes*, which are defined as those processes which have $\mathbf{\Lambda}(s)$ diagonal for all values of s. These processes include paper machines, adhesive coaters, polymer film extruders, and certain classes of distribution networks, such as those used in electric power systems and ship communication systems (Featherstone, 1995). The pseudo-SVD process structure was first most clearly defined by Hovd et al. (1993, 1996), who showed that controllers of the form $\mathbf{K}(s) = \mathbf{V}^{\mathrm{T}}\mathbf{\Lambda}_{\mathrm{K}}(s)$, $\mathbf{U}$ provided optimal stability and performance robustness for a wide variety of uncertainty structures. A list of manuscripts describing related approaches are cited by Hovd et al. (1993, 1996), the most closely related of which are discussed in sections 3.3 and 5.

In this next section, Lemma 1 is generalized to the multivariable case in terms of pseudo-singular values.

3. Model Requirements

In the following it is shown that to control a process with zero steady-state error it is necessary to correctly identify the signs of the pseudo-singular values. This result is shown using two approaches, each of which provides its own insights.

3.1. Steady-State Controllability and Integral Stability. Internal model control (IMC) is based on the Youla parametrization and for stable processes is a convenient framework for studying the limitations on achievable performance posed by time delays and right half-plane zeros (Braatz, 1995; Brosilow and Markale, 1992; Brosilow, 1979; Garcia and Morari, 1982, 1985a,b; Morari and Zafiriou, 1989). An IMC controller with a type I filter is a model-based controller with integral action. Controllers which maintain closed-loop stability with arbitrary detuning can be tuned on-line to compensate for time-varying disturbances without drastic consequences. The following lemma (proof in Appendix) indicates that the signs of all the pseudo-singular values must be correctly identified for a model-based integral IMC controller that maintains stability with arbitrary detuning to stabilize the true process.

Lemma 2. *Assume that the true plant* $\mathbf{P}_t(s)$ *and the plant model* $\mathbf{P}_m(s)$ *are strictly stable proper rational transfer functions that have the same steady-state rotation matrices* ($\mathbf{P}_t(0) = \mathbf{U}\boldsymbol{\Lambda}_t(0)\,\mathbf{V}^{\mathrm{T}}$ *and* $\mathbf{P}_m(0) = \mathbf{U}\boldsymbol{\Lambda}_m(0)\,\mathbf{V}^T$). *Then* $\Lambda_{m,ii}(0)/\Lambda_{t,ii}(0) > 0$, *for all i, if and only if there exists an IMC controller with a type I filter which stabilizes both* $\mathbf{P}_m(s)$ *and* $\mathbf{P}_t(s)$ *and maintains stability with arbitrary detuning.*

Lemma 2 generalizes Lemma 1 by assuming that the model has the same input rotation matrix **U** and output rotation matrix **V** as the true plant. This will only be true for special classes of processes, such as circulant symmetric processes (Featherstone and Braatz, 1995; Hovd et al., 1993). For general processes, Lemma 2 provides a necessary condition that a model-based integral IMC controller will stabilize the true process with some reliability. More specifically, if the sign of *any* pseudo-singular value is not known with confidence, then Lemma 2 implies that a stabilizing integral IMC controller cannot be designed *even if there were no errors in* ***U*** *and* ***V*** *and the controller was arbitrarily tuned.* Since there will in general be errors in **U** and **V**, an integral IMC controller designed based on a model with pseudo-singular values of incorrect sign would only stabilize the process by chance.

Section 3.2 shows that incorrectly identifying the sign of any pseudo-singular will lead to poor closed-loop performance, irrespective of the controller design. Also, section 3.2 delineates exactly which process input and output directions are controllable.

3.2. Controllability and Steady-State Performance. The importance of correctly identifying the sign of each pseudo-singular value is illustrated by considering the steady-state performance of a quiescent control system.

The true steady-state process gain matrix $\mathbf{P}_t(0)$ can be written in terms of the pseudo-singular values (via (2)):

$$\mathbf{P}_t(0) = \sum_{i=1}^{n} \Lambda_{t,ii}(0) U^i (V^i)^{\mathrm{T}} \tag{4}$$

Since the columns of **V** form a basis, the steady-state value of the manipulated variable u can be written as

$$u = \sum_{j=1}^{n} \alpha_j V^j \tag{5}$$

where the real scalar $\alpha_j = (V^j)^{\mathrm{T}}\, u$ quantifies the extent of manipulated variable movement u in the direction of V^j. Similarly, the effect of the disturbances d on the output (see Figure 1) can be written as

$$d = \sum_{j=1}^{n} \beta_j U^j \tag{6}$$

where the real scalar $\beta_j = (U^j)^{\mathrm{T}}\, d$ quantifies the extent of the steady-state disturbance d in the direction of U^j.

Thus the steady-state controlled output y is given by

$$y = \mathbf{P}_t(0)\, u + d \tag{7}$$

$$= \sum_{i=1}^{n} \Lambda_{t,ii}(0)\, U^i (V^i)^{\mathrm{T}} \sum_{j=1}^{n} \alpha_j V^j + \sum_{j=1}^{n} \beta_j U^j \tag{8}$$

$$= \sum_{j=1}^{n} (\Lambda_{t,jj}(0)\, \alpha_j + \beta_j) U^j \tag{9}$$

The projection of y in the direction of U^j is given by

$$(U^j)^{\mathrm{T}}\, y = \Lambda_{t,jj}(0)\, \alpha_j + \beta_j \tag{10}$$

A conclusion of (6) and (10) is that no controller can suppress steady-state output disturbances that are in the directions of the U^j corresponding to zero pseudo-singular values ($\Lambda_{t,jj}(0) = 0$). Since the analysis is in terms of the control action (rather than the controller), this argument holds for any controller, no matter how advanced (including model predictive, adaptive, and/or nonlinear). A conclusion of (5) and (10) is that manipulated variable movements in the directions of the corresponding V^j have no effect on the process output. For a controller to provide zero steady-state error in the projection of y in the direction of U^j, it must provide a u such that $\alpha_j = -\beta_j/\Lambda_{t,jj}(0)$. If the controller uses the incorrect sign of the pseudo-singular value $\Lambda_{t,jj}(0)$ in its control calculations, then $(U^j)^{\mathrm{T}}\, y$ in (10) will be greater than β_j, that is, closed-loop performance of y in the direction of U^j which is worse than the open-loop performance.

An interesting point concerning the ability to identify pseudo-singular values also follows directly from (6), (9), and (10), where d is now treated as being stochastic measurement noise rather than the effect of the steady-state disturbances on the output. When $\Lambda_{t,jj}(0)$ is small, then the response of y to the projection of u in the direction of V^j is small, so that the signal-to-noise ratio (the ratio of $(U^j)^{\mathrm{T}}\, y$ to β_j) is small. It is these small signal-to-noise ratios associated with the small pseudo-singular values that make the identification of large-scale processes challenging.

3.3. Comparison to Singular Values and Eigenvalues. The results in section 3.2 are related to results derived for singular values (Lau et al., 1985; Morari, 1983; Skogestad and Morari, 1987). The main point in these papers was that processes with small singular values require large manipulated variable moves for adequate disturbance suppression. As the pseudo-singular values have the same magnitude as the singular values, this point also holds for pseudo-singular values that are small in magnitude. On the other hand, the pseudo-singular values provide a more convenient multivariable generalization of the SISO concept of the need to "correctly identify the sign of the steady-state gain" than provided by singular values.

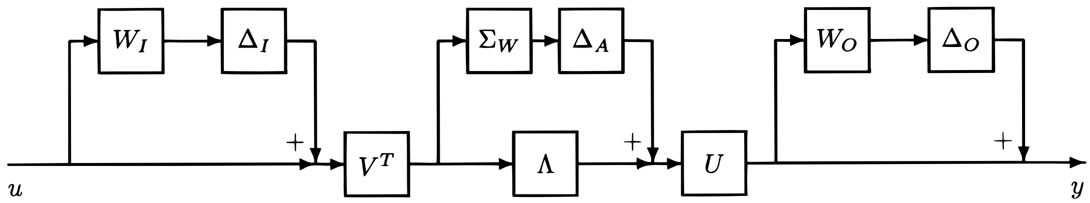

Figure 2. Block diagram of the process with input, output, and pseudo-singular values uncertainties.

Table 1. Data-Driven Multivariable Uncertainty Identification Algorithm

1. Perform least-squares identification of $\mathbf{P}(0)$.
2. Compute the pseudo-SVD of $\mathbf{P}(0)$.
3. Compute confidence intervals on the pseudo-singular values, assuming that $\mathbf{U}$ and $\mathbf{V}$ are fixed, and use these to define an additive uncertainty description.
4. Determine the number of plant directions that cannot be controlled with confidence by either (a) using the confidence intervals computed in step 3 or (b) applying a Monte Carlo approach.
5. Take errors in $\mathbf{U}$ and $\mathbf{V}$ into account using a multiplicative input and output norm-bounded uncertainty description.

Lemma 2 is closely related to a result derived for eigenvalues of the steady-state plant gain matrix multiplied by the inverse of the steady-state plant model (Garcia and Morari, 1985a; Koung and MacGregor, 1993, 1994). The main point of these papers was that the sign of these eigenvalues must be correctly identified before the process can be controlled with integral action. Since the true plant is never known, a major weakness of the eigenvalue conditions is that their coupling of the model and the plant makes it cumbersome to construct a data-driven uncertainty identification procedure for use in robust control. The weakness of eigenvalue conditions in general is that such conditions can lead to misleading indications of stability robustness (Doyle and Stein, 1981). A more appropriate framework for robustness analysis is in terms of singular values (Doyle, 1982; Doyle and Stein, 1981; Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996), for which the pseudo-singular values are equivalently suited as their magnitudes are the same as those for the singular values. The pseudo-singular values blend the strengths of both singular value and eigenvalue analyses.

4. Multivariable Uncertainty Identification

It was shown in section 3 that a model-based controller implemented on a process for which the sign of any of the pseudo-singular values is incorrectly identified will provide poor performance, irrespective of how the controller is designed. Lemma 2 provides strong evidence that such a controller, if no constraints were present, would likely provide an unstable closed-loop response. Due to the constraints on the manipulated variables which always exist in practice, the controller either will cause the manipulated variables to drift until the constraints are hit or will induce bounded oscillations. Both behaviors have been observed in simulations (Featherstone, 1997) and would pose a significant problem in an industrial control system. In either case, there is an amplification of disturbances in directions associated with the misidentified pseudo-singular values.

This motivates the development of algorithms to quantify the accuracy of the pseudo-singular values from input−output data, which can be used to define a nonconservative uncertainty description that incorporates the inaccuracies in the pseudo-singular values as well as represent inaccuracies in the input and output rotation matrices (see Figure 2). This is most closely related to an uncertainty description proposed by Koung and MacGregor (1994). The most significant difference is that we propose to use norm-bounded uncertainty descriptions (for details on robust control, see the literature (Doyle, 1982; Morari and Zafiriou, 1989; Safonov, 1982; Skogestad and Postlethwaite, 1996)) for the input and output rotation matrices rather than the element-by-element uncertainty descriptions described by Koung and MacGregor (1994). Norm-bounded uncertainty descriptions are more accepted in the robust control community while element-by-element descriptions are too conservative for use in quantitative worst-case robustness analysis (Morari and Zafiriou, 1989).

The proposed data-driven multivariable uncertainty identification algorithm consists of the five steps described in Table 1. Each step of the algorithm is described in more detail below.

Computation of Confidence Intervals (Steps 1−3 in Table 1). For step 1, it is assumed that $\mathbf{P}(0)$ is linear in the parameter vector β. This is always true for black-box models. For phenomenological models which may have $\mathbf{P}(0)$ nonlinear in the parameters, use the linearization of $\mathbf{P}(0)$ with respect to β, which is an accepted approximation in the parameter estimation literature (Beck and Arnold, 1977). For brevity, we will treat only the case where the steady-state matrix is computed from step response experiments, as the generalization for arbitrary input−output experiments is straightforward (see Ljung (1987)).

At the kth experiment with input u^k, the measured output is

$$y_{\mathrm{m}}^k = \mathbf{P}(0)\, u^k + \epsilon^k = \mathbf{X}(u^k)\, \beta + \epsilon^k \tag{11}$$

where ϵ^k is the effect of unmeasured disturbances on the measured output and $\mathbf{X}(u^k)$ is referred to as the *input* matrix (Beck and Arnold, 1977), since it depends on the experimental inputs as well as the model structure. For brevity, ϵ^k is assumed to be independent in time and location with known variance σ^2 (generalizing to the case of temporally-correlated disturbances and unknown variance is straightforward, e.g., see Appendix II of (Ljung, 1987)).

For N experiments, the measurements can be stacked to give

$$Y_{\mathrm{m}} = \begin{bmatrix} y_{\mathrm{m}}^1 \\ \vdots \\ y_{\mathrm{m}}^N \end{bmatrix} = \begin{bmatrix} \mathbf{X}(u^1) \\ \vdots \\ \mathbf{X}(u^N) \end{bmatrix} \beta + \begin{bmatrix} \epsilon^1 \\ \vdots \\ \epsilon^N \end{bmatrix} = \mathbf{X}\beta + \epsilon \tag{12}$$

Then the least-squares estimate and the covariance

Table 2. Monte Carlo Algorithm To Compute the Number of *q* Directions That Are Uncontrollable[a]

1. Treat the steady-state plant matrix from step 1 in Table 1 as equal to the true plant matrix and use the assumed level of noise (the noise model can also be computed directly from the data (Beck and Arnold, 1977)) and the manipulated variable moves used to construct a large number of k of simulated output data files.
2. For each of the k data files, apply least squares to compute a steady-state model gain matrix $\mathbf{P}_{m,j(0)}$, $j = 1, ..., k$.
3. For each of the data files j, compute $\gamma_{i,j} = Re\{\lambda_i(\mathbf{P}_t(0)\mathbf{P}_{m,j}^{-1}(0))\}$ for all $i = 1, ..., n$.
4. Record the number of $\gamma_{i,j}$ less than zero for each plant model j (called this number q_j) and collectively use this to define a discrete probability distribution function for q.
5. Select q based on the discrete probability distribution function so that there is $100(1 - \rho)\%$ confidence that $n - q$ directions are controllable.

[a] The criterion in step 3 is based on Theorem 2 of (Garcia and Morari, 1985a).

matrix of the parameters are

$$\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T Y_m; \qquad \mathrm{cov}(\hat{\beta}) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1} \quad (13)$$

From the model of the steady-state interaction matrix $\mathbf{P}_m(0)$ obtained from $\hat{\beta}$, step 2 consists of computing the pseudo-SVD as described in section 2.

From the computed **U** and **V** via (2), the ith pseudo-singular value is

$$\Lambda_{m,ii}(0) = (U^i)^T \mathbf{P}_m(0)\, V^i = (U^i)^T \mathbf{X}(V^i)\, \hat{\beta} \quad (14)$$

where $\mathbf{X}(V^i)$ has the same form as $\mathbf{X}(u_k)$ in (11) but is now calculated with V^i. The $\Lambda_{m,ii}(0)$ can be stacked into a vector $\lambda_m(0)$ which is given by

$$\lambda_m(0) = \mathbf{X}_{\mathbf{U},\mathbf{V}}\hat{\beta} \quad (15)$$

where $\mathbf{X}_{\mathbf{U},\mathbf{V}}$ is the linear transformation between $\hat{\beta}$ and $\lambda_m(0)$. With this transformation, the covariance matrix for $\lambda_m(0)$ is given by

$$\mathrm{cov}(\lambda_m(0)) = \mathbf{X}_{\mathbf{U},\mathbf{V}}\mathrm{cov}(\hat{\beta})\mathbf{X}_{\mathbf{U},\mathbf{V}}^T = \sigma^2\mathbf{X}_{\mathbf{U},\mathbf{V}}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}_{\mathbf{U},\mathbf{V}}^T \quad (16)$$

where **X** is from (12). With the covariance matrix for $\lambda_m(0)$, a $100(1 - \rho)\%$ confidence interval for each pseudo-singular value $\Lambda_{m,ii}(0)$, with variance σ_i, is given by

$$(\Lambda_{m,ii}(0) - z_{\rho/2}\sigma_i,\ \Lambda_{m,ii}(0) + z_{\rho/2}\sigma_i) \quad (17)$$

where values of the standard normal deviate $z_{\rho/2}$, at various levels of significance can be calculated or referenced from statistical tables (Beck and Arnold, 1977; Devore, 1982). These confidence intervals are used to define a worst-case additive uncertainty description (see Figure 2). The uncertainty matrix $\mathbf{\Delta}_A$ for the pseudo-singular value estimates is diagonal, with the weighting matrix $\mathbf{\Sigma}_W$ defined by the radii of the confidence intervals.

Determining the Number of Controllable Directions (Step 4 in Table 1). Steps 4a and 4b provide alternative approaches for determining the number of plant directions that can be controlled with confidence. Step 4a is based on the confidence intervals computed in step 3. If step 3 provides a confidence interval for a pseudo-singular value that includes zero, then the linear statistics indicates that the sign is not known with confidence (although not shown here for brevity, this can be posed rigorously in terms of hypothesis testing, as discussed in detail in Chapter 5 of (Featherstone, 1997)). While step 4a provides a simple analytical approach for computing the controllable directions, step 4b uses a Monte Carlo approach to provide an improved estimate on the number of controllable directions. The Monte Carlo method described in Table 2 accounts for errors in **U** and **V** when determining the number q of plant directions that cannot be controlled with confidence. Since the signal-to-noise ratios are the poorest for the smallest pseudo-singular values (as discussed in section 3), this suggests that the smallest q pseudo-singular values should not be controlled.

Theoretically, both steps 4a and 4b provide optimistic (that is, underestimated) estimates on the number of controlled directions. Step 4a is optimistic because the bounds on the pseudo-singular values from step 3 are underestimated, since errors in **U** and **V** are not explicitly taken into account. Step 4b is optimistic because it uses the steady-state plant model in place of the true plant in its computations (this must be done because the true plant is not known in practice). In our experience (see section 6 and (Featherstone, 1997)), step 4b is less optimistic than step 4a.

Theoretically, these optimistic estimates would result in too few pseudo-singular values being classified as being uncontrollable. These optimistic estimates do not pose any problems as long as a high level of confidence is selected for definition of the confidence interval in (17). This is illustrated in section 6.

Uncertainty Description for Input and Output Rotation Matrices (Step 5 in Table 1). Steps 1−4 (in Table 1) are sufficient for computing the number of controllable directions and estimating the inaccuracies associated with these directions but does not account for inaccuracies in the input and output rotation matrices. Here we provide some guidelines for constructing input and output uncertainty descriptions that are intended to be helpful for industrial practitioners.

The number of independent degrees of freedom in **U** and **V** is $n^2 - n$. Identifying these elements to the high level of accuracy required for an element-by-element uncertainty description to be nonconservative (Morari and Zafiriou, 1989) would require a large quantity of high-quality experimental data, typically much more than what would be available in practice. Hence, statistics on the individual elements of **U** and **V** are not suitable for developing a worst-case multivariable uncertainty description for the input and output rotation matrices. Instead, we suggest using a norm-bounded uncertainty description (Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996) for **U** and **V**.

The uncertainty associated with each manipulated variable (which could be either an actuator or the setpoint to a lower level control loop, such as a flow control loop (Ogunnaike and Ray, 1994)) is normally assumed to be independent of the other manipulated variables, which corresponds to a diagonal perturbation block Δ_I (Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996). Representing Δ_I as being full, however, accounts for arbitrary (though bounded)

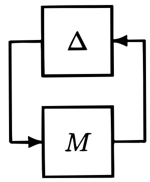

Figure 3. **M**−**Δ** block structure.

rotational inaccuracies in the input rotation matrix **V**. An appropriate uncertainty weight would typically allow for 10% steady-state error in manipulated variable movements and 150−200% error at high frequencies (Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996); this leaves only 1 degree of freedom in the uncertainty weight, which is the turning point. The turning point indicates the smallest frequency at which the steady-state uncertainty description in no longer adequate for representing the plant/model mismatch. A simple way to select this is based on the frequency at which each manipulated variable's frequency response starts to become less reproducible (this could be quantified by performing several frequency response identification tests on each manipulated variable).

If an estimate of typical sensor biases are available, then these can be used to define the weights on the output uncertainty. Since the robustness of closed-loop multivariable systems is much less sensitive to output uncertainties than input uncertainties (Koung and MacGregor, 1993; Skogestad et al., 1988; Skogestad and Postlethwaite, 1996), the selection of this weight is usually not critical. Assuming the output uncertainty to be full-block accounts for arbitrary (though bounded) rotational inaccuracies in **U**.

Robustness Analysis. The robust stability requirement corresponds to the block diagram for the process with uncertainty blocks as shown in Figure 2 (see (Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996) for background on the use of μ for robustness analysis). To analyze the robust stability of the uncertain process under feedback control, compute μ of the following $\mathbf{M}(s)$ matrix (as shown in Figure 3) constructed from the block diagram of the process (Figure 2) in a closed loop:

$$\mathbf{M}(s) = \begin{bmatrix} -\Sigma_W V^T K(I+PK)^{-1} U & -\Sigma_W V^T K(I+PK)^{-1} & \Sigma_W V^T (I+KP)^{-1} \\ W_O (I+PK)^{-1} U & -W_O PK(I+PK)^{-1} & W_O P(I+KP)^{-1} \\ -W_I K(I+PK)^{-1} U & -W_I K(I+PK)^{-1} & -W_I KP(I+KP)^{-1} \end{bmatrix} \quad (18)$$

with $\mathbf{\Delta} = \mathrm{diag}\{\Delta_A, \Delta_O, \Delta_I\}$.

5. An Appropriate Robust Controller Structure

In section 3 it was shown that the controller must not manipulate in directions associated with a pseudo-singular value whose sign is not reliably known, as such manipulations would lead to poor performance. This motivates the development of a controller structure that cannot perform manipulations in these directions. Define $\tilde{\mathbf{\Lambda}}(s)$ as the $(n-q) \times (n-q)$ upper left submatrix of the pseudo-singular value matrix $\mathbf{\Lambda}(s)$ defined in (3). The input directions of this portion of the plant are controllable. Define the *pseudo-SVD controller structure* as

$$\mathbf{K}(s) = \mathbf{V}\begin{bmatrix} \mathbf{\Lambda}_K(s) & 0 \\ 0 & 0 \end{bmatrix}\mathbf{U}^T \quad (19)$$

where **U** and **V** are defined in (2), and $\mathbf{\Lambda}_K(s)$ is a $(n-q) \times (n-q)$ transfer function matrix (not necessarily diagonal). An additional motivation for this controller structure is that, when the process has a diagonal $\mathbf{\Lambda}(s)$ in (3), the controller structure (19) provides optimal stability and performance robustness for a wide variety of uncertainty structures, including the one in Figure 2 (Hovd et al., 1993, 1996). Several control engineers over the past 20 years have proposed controllers of a related form based on the singular value decomposition of the steady-state gain matrix (Downs and Moore, 1981; Lau et al., 1985; MacFarlane and Kouvaritakis, 1977; Moore, 1986). A significant difference here is that the purpose of the controller structure in (19) is to prevent the controller $\mathbf{K}(s)$ from manipulating in uncontrollable directions, while the objective of earlier work was to design decouplers (Downs and Moore, 1981; Hung and MacFarlane, 1982; Lau et al., 1985; MacFarlane and Kouvaritakis, 1977; Moore, 1986). As such, while $\Lambda_K(s)$ is restricted to be diagonal as in earlier work (Downs and Moore, 1981; Hung and MacFarlane, 1982; Lau et al., 1985; MacFarlane and Kouvaritakis, 1977; Moore, 1986), $\mathbf{\Lambda}_K(s)$ *is not restricted to be diagonal in (19).* In fact, the matrix $\mathbf{\Lambda}_K(s)$ can be designed to control $\tilde{\mathbf{\Lambda}}(s)$ using any *multivariable* controller design technique, e.g., linear quadratic control, μ-optimal control (Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996).

By allowing $\mathbf{\Lambda}_K(s)$ to be nondiagonal, there is no restriction that the input and output rotation matrices of the process be constant (or approximately constant), as is implicitly assumed when $\mathbf{\Lambda}_K(s)$ is restricted to be diagonal (Downs and Moore, 1981; Hung and MacFarlane, 1982; Lau et al., 1985; MacFarlane and Kouvaritakis, 1977; Moore, 1986). Our proposed controller structure makes the much less severe restriction that the controller should not manipulate in directions that are uncontrollable at steady state. This does not provide a performance limitation for most processes, since a plant direction that is uncontrollable at steady state is most likely to be uncontrollable at higher frequencies as well. On the other hand, the proposed controller structure may reduce the achievable performance for process models that are significantly more accurate at intermediate frequencies than at low frequencies.

Regardless of the desired controller design procedure, either weights or filter parameters in the design procedure can be selected so that the controller is robustly stable to the uncertainty in the pseudo-singular values as well as in the input and output rotation matrices **U** and **V** (using (18)). The application of the uncertainty identification procedure of section 4 and the controller structure (19) is demonstrated next.

6. Simulation Example for an Industrial Paper Machine

The key concepts are illustrated using a problem description patterned after a paper machine model reported in the industrial process control literature. The interaction matrix for a paper machine without edge effects is given by (the superscript stands for "*w*ithout *e*dge effects")

$$\mathbf{P}^{\mathrm{we}}(s) = \frac{e^{-\theta s}}{\tau s + 1} \underbrace{\begin{bmatrix} p_1 & p_2 & \cdots & p_{10} & 0 & \cdots & \cdots & 0 \\ p_2 & p_1 & p_2 & \cdots & p_{10} & \ddots & \cdots & \vdots \\ \vdots & p_2 & p_1 & p_2 & \cdots & \ddots & \ddots & \vdots \\ p_{10} & \vdots & p_2 & \ddots & \ddots & \vdots & p_{10} & 0 \\ 0 & p_{10} & \vdots & \ddots & \ddots & p_2 & \vdots & p_{10} \\ \vdots & \ddots & \ddots & \cdots & p_2 & p_1 & p_2 & \vdots \\ \vdots & \ddots & \ddots & p_{10} & \cdots & p_2 & p_1 & p_2 \\ 0 & \cdots & \cdots & 0 & p_{10} & \cdots & p_2 & p_1 \end{bmatrix}}_{101 \times 101} \quad (20)$$

with

$$p_1 = 1;\ p_2 = 0.9;\ p_3 = 0.7;\ p_4 = 0.8;\ p_5 = 1;\ p_6 = 0.6;\ p_7 = -0.5;\ p_8 = -0.4;\ p_9 = -0.2;\ p_{10}\ -0.2;\ \theta = \tau = 1 \quad (21)$$

These interaction parameters are for a paper board machine described by Karlsson and Haglund (1983).

The actuators near the edges are assumed to have a reduced effect on all corresponding downstream sensor lanes (the superscript stands for "*e*dge effects"):

$$P^{\mathrm{e}}(i,1) = 0.5P^{\mathrm{we}}(i,1);\quad \forall i = 1, ..., n$$

$$P^{\mathrm{e}}(i,2) = 0.75P^{\mathrm{we}}(i,2);\quad \forall i = 1, ..., n$$

$$P^{\mathrm{e}}(i,100) = 0.75P^{\mathrm{we}}(i,100);\quad \forall i = 1, ..., n$$

$$P^{\mathrm{e}}(i,101) = 0.5P^{\mathrm{we}}(i,101);\quad \forall i = 1, ..., n \quad (22)$$

where $A(i,j)$ refers to the ith row and jth column of $\mathbf{A}$.

Identification. During identification, the measured process output at steady state y_{m} is assumed to be given by

$$y_{\mathrm{m}} = \mathbf{P}_{\mathrm{t}}(0)u + \epsilon \quad (23)$$

where u is the actuator input move and ϵ represents zero-mean Gaussian measurement noise. In the standard industrial experiment (called a "bump test"), the open-loop response is measured for a step in several manipulated variables across the machine (Heaven et al., 1993). This defines the process input as $u = e_{10} + e_{35} + e_{60} + e_{85}$, where

$$e_k^{\mathrm{T}} = [0\ \ \ldots\ \ 0\ \ 1\ \ 0\ \ \ldots\ \ 0]^{\mathrm{T}} \quad (24)$$

with the 1 in the kth position.

The measurement noise in each sensing location is considered to be independent and to have a variance of 0.04 (this is a reasonable value for many paper grades). Since it is industrial practice to repeat the bump tests several times to reduce the effects of noise, the simulated identification experiments consisted of five step input tests. The process model interaction matrix was assumed to have the structure of $\mathbf{P}^{\mathrm{we}}$ in (20), while the true process interaction matrix $\mathbf{P}^{\mathrm{e}}$ is defined by (21) and (22). The estimated parameters $\hat{p}_j$ were calculated from the input−output data using least squares (step 1 in Table 1).

The next step is to determine which signs of the identified pseudo-singular values are known with confidence, based on the experimental data. The confidence interval of each pseudo-singular value is calculated

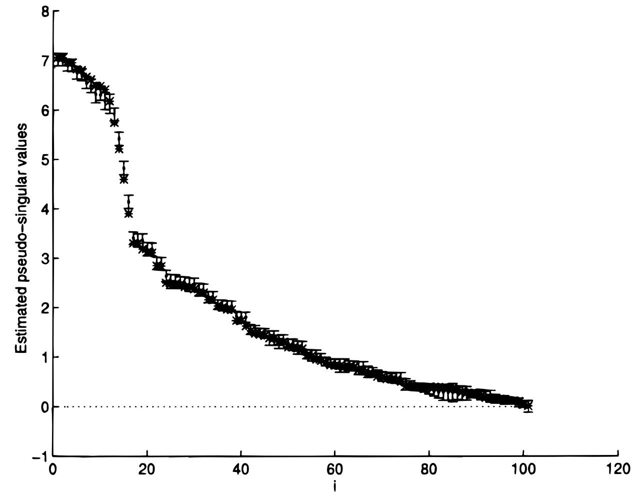

Figure 4. 95% confidence intervals for each pseudo-singular value $\Lambda_{\mathrm{m},ii}(0)$.

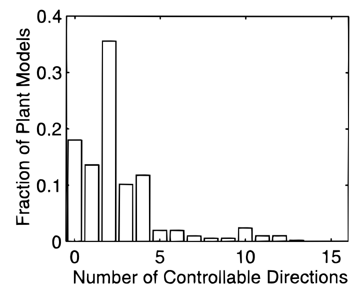

Figure 5. Fraction of estimated plant models that have a specified number of misidentified pseudo-singular values $\Lambda_{\mathrm{m},ii}(0)$, where the first nominal model is used in place of the true plant. Five hundred experimental data sets were used in the Monte Carlo simulation.

using the estimated noise variance (this is known in practice or can be estimated from the data (Beck and Arnold, 1977)), the number of bump tests, the known input u, and the measured output y_{m} (steps 2 and 3 in Table 1). Figure 4 shows the 95% confidence interval for each pseudo-singular value of the estimated model (this corresponds to a 97.5% hypothesis test). The intervals for the last three pseudo-singular value estimates include zero, indicating that these signs are not known with 97.5% confidence (step 4a). In fact, one of the signs of the pseudo-singular values was identified incorrectly.

On the other hand, step 4b estimates that there is 77% confidence that less than four pseudo-singular values have incorrect signs (see Figure 5). Recall from section 4 that both steps provide optimistic estimates on the number of controllable directions, with step 4b being less optimistic than step 4a. To determine how optimistic each step is, Monte Carlo simulations were performed using the true plant (see Figure 6). There is 59% confidence that less than four pseudo-singular values have incorrect signs, with 85% confidence that less than six pseudo-singular values have incorrect signs.

Another interesting point can be made from Figure 6. The probability that a single nominal model has all of its directions controllable is less than 3%. Based on the results of section 3, this implies that a model-based controller that attempts to control all of the plant directions has only a 3% probability of providing acceptable closed-loop performance.

The uncertainty description was constructed as described in section 4. The additive uncertainty weights

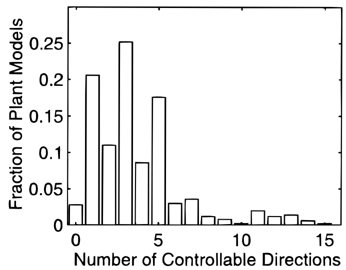

Figure 6. Fraction of estimated plant models that have a given number of misidentified pseudo-singular values $\Lambda_{m,ii}(0)$, where the true process is used. Five hundred experimental data sets were used in the Monte Carlo simulation.

were based on the confidence intervals in Figure 4. The input uncertainty weight $W_I(s) = 0.3[(0.1s + 1)/(0.02s + 3)]I$ allows for 10% steady-state error in manipulated variable movements and 150% error at high frequencies, with turning points at $\omega = 10$ and $\omega = 150$. The output uncertainty was selected to be the same weight: $W_O = W_I$. These weights also account for uncertainties in the scalar dynamics.

Controller Synthesis. The control objective minimizes the effect of output disturbances d on the controlled variable y (see Figure 1), while being robustly stable to input, output, and pseudo-singular value uncertainties. For this process, it is natural to select $\mathbf{\Lambda}_K(s)$ in (19) as being diagonal. Each diagonal element $\Lambda_{K,ii}(s)$ of the pseudo-SVD controller (19) can be designed by any controller method; we decided to use multiloop IMC-PID tuning (Morari and Zafiriou, 1989):

$$\Lambda_{K,ii}(s) = \frac{1}{\Lambda_{m,ii}(0)} \frac{\left(1 + \tau_D s + \frac{1}{\tau_I s}\right)}{\tau_{F,i} s + 1} \frac{2\tau + \theta}{2(\lambda_i + \theta)}, \quad \forall i = 1, ..., n - q \qquad (25)$$

with

$$\tau_I = \tau + \theta/2; \qquad \tau_D = \frac{\tau\theta}{2\tau + \theta}; \qquad \tau_{F,i} = \frac{\lambda_i \theta}{2(\lambda_i + \theta)} \qquad (26)$$

where λ_i is the IMC tuning parameter. The SISO controllers $\Lambda_{K,ii}(s)$ are stacked up as the diagonal elements of a matrix $\mathbf{\Lambda}_K(s)$, with the overall controller computed from (19).

Based on the identification results, nonzero SISO controllers $\Lambda_{K,ii}(s)$ are designed based on the reliably identified pseudo-singular values. The IMC parameter λ_i for each SISO controller $\Lambda_{K,ii}(s)$ was tuned as fast as possible while achieving robust stability ($\mu(M) < 1$); thus, the closed-loop system is stable to input, output, and pseudo-singular value variations.

Time Domain Solutions. The controller with IMC tuning parameters is compared to a model-inverse-based controller, which has been implemented on many paper machines (as surveyed by (Braatz et al., 1996)):

$$\mathbf{K}(s) = k(s)[\mathbf{P}_m(s)]^{-1} = k(s)\, \mathbf{V}\mathbf{\Lambda}_m^{-1}(s)\, \mathbf{U}^T \qquad (27)$$

with

$$k(s) = k_c\left(1 + \frac{1}{\tau_I s}\right); \qquad k_c = \frac{2\tau + \theta}{2\lambda}; \qquad \tau_I = \tau + \frac{\theta}{2} \qquad (28)$$

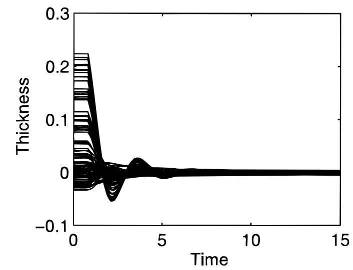

Figure 7. Profile response using the pseudo-SVD controller.

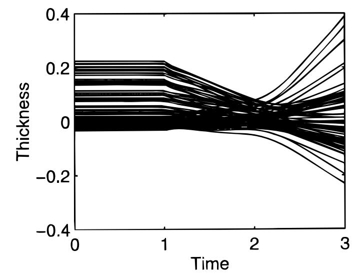

Figure 8. Profile response using the model-inverse-based controller.

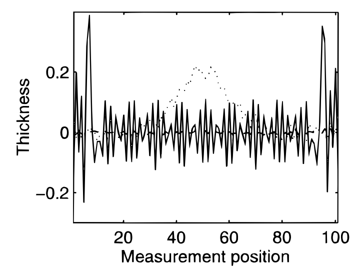

Figure 9. Initial disturbance profile (···), profile at $t = 3$ for pseudo-SVD controller (− −), and profile at $t = 3$ for model-inverse-based controller (—).

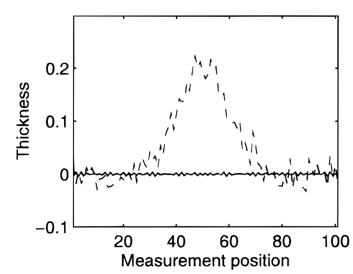

Figure 10. Initial disturbance profile (− −) and the steady-state profile response using the pseudo-SVD controller (—).

and $\lambda = 2\theta$, which is within the common recommendation (Morari and Zafiriou, 1989).

Figures 7 and 8 show the closed-loop response to the initial disturbance profile shown in Figure 9 using the pseudo-SVD and model-inverse-based controllers, respectively. Figures 7 and 10 reveal that, although the response for the pseudo-SVD controller nearly goes to zero, some offset remains at steady state due to the projection of the disturbance in the uncontrolled directions. In comparison, the response using the model-inverse-based controller reveals that, while a portion of the disturbance is initially reduced, the directions corresponding to misidentified pseudo-singular values eventually take over, causing the controlled variables to drift. Figure 9 shows the profiles across the machine for the pseudo-SVD and model-inverse-based controllers at time $t = 3$. In terms of average variation across the paper machine, the profile for the model-inverse-based controller at this time step is already worse than if the process had been left in open loop. Such closed-loop behavior has been observed in a large number of industrial paper machines (Bialkowski, 1986).

7. Interaction between Design and Control

The focus of this paper was the application of the pseudo-singular values to the integrated robust identification and control of large-scale processes. The pseudo-singular values can also be used to assess controllability at the process design stage (this is the most common use of controllability analysis measures). A promising approach would be to incorporate their use into the methodology of Perkins and co-workers (Narraway et al., 1991; Narraway and Perkins, 1993, 1994; Perkins and Walsh, 1994; Walsh and Perkins, 1994), which assesses the effect of disturbances on the overall process economics. For each design, the identification procedure (in section 4) would produce the model uncertainty description for use in the Perkins procedure (Perkins and Walsh, 1994; Walsh and Perkins, 1994), with the cost of the identification experiments added to their economic cost function. If only a steady-state simulation model was available, then (10) could be applied to compute the effect of the disturbances on the process output, which would be used by the Perkins procedure to quantify the economic losses due to the disturbances.

8. Conclusions

One of the main limitations of robust control has been the lack of identification techniques that provide nonconservative uncertainty descriptions for multivariable processes. We have shown that it is absolutely critical that the signs of the pseudo-singular values of the model be known with confidence, as only the associated process directions can be reliably controlled. A practically-motivated uncertainty structure was proposed that takes into account uncertainty in the pseudo-singular values, the plant input rotation matrix, and the plant output rotation matrix. Some guidelines were given for quantifying the accuracy of the input and output rotation matrices. The accuracy of the pseudo-singular values was quantified from input–output data, and two algorithms were proposed for determining the number of controllable plant directions.

A robust controller structure was proposed which manipulates only in the controllable plant directions. Any reasonable controller design technique can be applied within the proposed controller structure to provide robust stability to inaccuracies in the controlled pseudo-singular values and in the input and output rotation matrices. The applicability of the proposed robust identification and control algorithms was demonstrated on a simulation example patterned after an industrial paper machine. Poor behavior similar to that seen in industry was observed when attempting to control in all plant directions. The proposed controller structure provided reliable performance.

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Appendix

Proof of Lemma 2. By Theorem 2 of Garcia and Morari (Garcia and Morari, 1985a), a stabilizing integral IMC controller exists if and only if $Re\{\lambda_i(\mathbf{P}_t(0)\mathbf{P}_m^{-1}(0))\} > 0$, for all $i = 1, ..., n$. With the steady-state decompositions of $\mathbf{P}_t$ and $\mathbf{P}_m$, this is equivalent to $Re\{\lambda_i(\mathbf{\Lambda}_t(0)\mathbf{\Lambda}_m^{-1}(0))\} > 0$, which is equivalent to $Re\{\Lambda_{m,ii}(0)/\Lambda_{t,ii}(0)\} > 0, \forall i = 1, ..., n$. This is equivalent to $\Lambda_{m,ii}(0)/\Lambda_{t,ii}(0) > 0, \forall i = 1, ..., n$ since the pseudo-singular values are real. □

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