Identification and Cross-Directional Control of Coating Processes

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Of special industrial interest is the cross-directional control of coating processes, where the cross direction refers to the direction perpendicular to the substrate movement. The objective of the controller is to maintain a uniform coating under unmeasured process disturbances. Assumptions that are relevant to coating processes found in industry are used to develop a model for control design. This model is used to derive a model predictive controller to maintain flat profiles of coating across the substrate by varying the liquid flows along the cross direction. Actuator constraints, measurement noise, model uncertainty, and the plant condition number are investigated to determine which of these limit the achievable closed-loop performance. From knowledge of how these limitations affect the performance we can make some recommendations on how to modify the plant design to improve the coating uniformity. The theory developed throughout the article is rigorously verified through experiments on a pilot plant. The controller rejects disturbances with two sampling times. The proposed controller can reduce the variance in coating thickness by as much as 80% compared to what is possible by manual control or simple control schemes.

Introduction

Coating refers to the coating of a substrate with a uniform layer of liquid. Coating processes are of great importance to manufacturing, especially in the photographic, magnetic and optical memory, electronic, adhesive, and paper industries (Cohen, 1990).

Plant description

Figure 1 is a simplified diagram of a typical plant. The process begins with a feed roller from which substrate is unwound. From there, the substrate passes between a roller and a stainless steel die. The liquid flows through a slot in the die to the substrate. The cavity in the die is designed to distribute a uniform flow of liquid through the slot. A controlled pump supplies a constant flow of liquid through the die.

The term "gap width" refers to the distance across the slot at a given point along the die. The gaps through which liquid flows are adjusted by means of n equally spaced bolts. The bolts are adjusted manually.

After being coated with liquid, the substrate passes through a drier. After the drier, the time-averaged coating thickness at each of the n positions corresponding to the die bolts is measured by a traversing coat-weight sensor. The coated substrate is wound on the product roller.

For further details on die design, die flow phenomena, drying phenomena, coat-weight sensors, and other aspects of coating, see Sartor (1990), Cohen (1990), Cohen et al. (1990), Scriven and Suszynski (1990), and the literature cited therein.

Control objective

The cross-directional control problem is aimed at maintaining a uniform profile of liquid across the substrate. Successful control of coating thickness improves product quality and reduces the time needed to bring the plant on-line. Poor control

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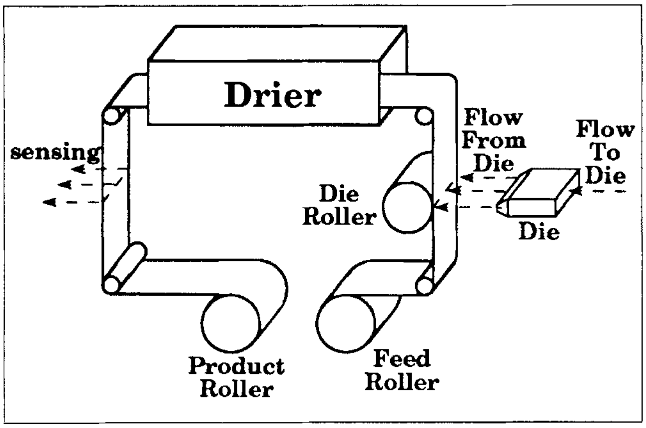

Figure 1. Typical coating plant.

can lead not only to coating thickness nonuniformity but also coating instabilities that leave portions of the substrate uncovered; such substrate must be rejected (for a short summary of coating instabilities, see Sartor, 1990).

We will consider coating processes with a large time delay between a change in gap width and the resulting sensing of the change in coating profile downstream. This time delay could be due to a sensor installed at a fair distance from the die as in the coating plant considered above. Because the controller cannot be expected to reject disturbances faster than this time delay, detailed process dynamics are not considered in the modeling, identification, and control of the cross-directional coating process. Thus, the objective of the controller is the elimination of slow disturbances in the coating thickness. The disturbances were of this nature in the Avery/Dennison pilot plant; the control of this plant is studied in this article.

Organization

Assumptions that are relevant to a subset of coating processes found in industry are used to develop a model for control design. This model is used to derive an unconstrained model-predictive controller to maintain flat profiles of liquid across the substrate by varying the gap widths. Several modifications to the unconstrained controller are proposed to prevent physically infeasible actuator movements (gap widths). The simplest, yet effective, constraint-handling method is chosen.

Actuator constraints, measurement noise, model uncertainty, and the plant condition number are investigated to determine which of these limit the achievable closed-loop performance. The theory developed throughout the article is applied to a pilot-plant liquid coating process at the Avery/Dennison Research Center in Pasadena.

Notation

All scalars are italicized. Matrices are upper case italic bold. $M_{i,j}$ represents the (i,j) element of the matrix $\boldsymbol{M}$. Vectors are lower case italic bold. The ith element of the vector $\boldsymbol{x}$ is represented as x_i. $\boldsymbol{x}(t)$ refers to the value of $\boldsymbol{x}$ at time t.

Model Development

Below we make assumptions on the plant that are relevant to a subset of coating processes found in industry. These assumptions are used to develop a dimensional model. This model is transformed to a dimensionless form. The dimensionless model is then rearranged into a form suitable for controller design.

Dimensional model

Consider a plant with the number of actuators n equal to the number of sensors (or sensor measurement positions). It has been found experimentally (through examination of pilot-plant data) that the plant behaves approximately linearly in the operating region. Let $\check{\boldsymbol{u}}$ be the vector of gap widths, $\check{\boldsymbol{x}}$ be the vector of coating thicknesses, and $\check{\boldsymbol{v}}$ collect any effects on the coating thickness not due to changes in gap width. If the process dynamics are approximated by a pure delay, then the coating thickness at sampling instant t is related to the gap width at the previous sampling instant through

$$\check{\boldsymbol{x}}(t) = \boldsymbol{P}\check{\boldsymbol{u}}(t-1) + \check{\boldsymbol{v}}(t), \tag{1}$$

where $\boldsymbol{P}$ is a constant $n \times n$ matrix.

Assumption on $\check{\boldsymbol{v}}$. It accounts for unmeasured input effects such as measurement noise and disturbances. We assume that $\check{\boldsymbol{v}}$ is a nonzero-mean stochastic variable, that is, $\{\check{\boldsymbol{v}}(0), \check{\boldsymbol{v}}(1), \ldots, \check{\boldsymbol{v}}(h), \ldots\}$ is a sequence of independent random vectors with nonzero mean (Ljung, 1987). We define the steady-state disturbance $\check{\boldsymbol{d}}$ as the time-averaged value of $\check{\boldsymbol{v}}$, and define $\check{\boldsymbol{n}}$ by

$$\check{\boldsymbol{n}}(t) = \check{\boldsymbol{v}}(t) - \check{\boldsymbol{d}}. \tag{2}$$

We will assume that $\check{\boldsymbol{n}}$ is white noise. It will be referred to as measurement noise.

$\check{\boldsymbol{v}}$ is chosen to be stochastic because it describes well the apparently random fluctuations of the process. In practice, equal gap widths do not give a uniform coating because of imperfections in the roller or the die, nonuniformities in the drying process, or poor calibration of the gap widths. These imperfections lead $\check{\boldsymbol{v}}$ to have nonzero mean.

Assumptions on $\boldsymbol{P}$. Typically, the total flow of coating through the die is maintained constant through a high gain controller. Because of constant total flow, increasing the flow through one actuator will necessitate decreasing the flow through the others. In the development of the model, we make the following assumptions:

1. The total liquid flow (and therefore the sum of the coating thicknesses) is constant.
2. The responses to all actuators are similar and symmetric about the actuator positions.
3. The only interactions between the actuators are due to the constant flow assumption.

Assumption 2 implies that $\boldsymbol{P}$ is symmetric. Assumption 3 implies that $\boldsymbol{P}$ can be separated into two matrices

$$\boldsymbol{P} = \check{k}\boldsymbol{I} - \boldsymbol{M}, \tag{3}$$

where $\check{k}$ is the gain between the ith gap width and its corresponding coating thickness for an infinitely wide die (that is, $n \rightarrow \infty$). $\boldsymbol{I}$ is the $n \times n$ identity matrix, $\check{k}\boldsymbol{I}$ is the contribution that changing gap widths would have on the coating thicknesses if there were no interactions, and $\boldsymbol{M}$ represents the effect that increasing one gap width has on decreasing the flow through all the gaps. Assumption 3 also implies that all elements of $\boldsymbol{M}$ are equal, that is, $M_{i,j}=m$ for $i, j=1, 2, \ldots, n$. Then

$$\boldsymbol{P}=\underbrace{\begin{bmatrix} \check{k}-m & -m & -m & \cdots & -m \\ -m & \check{k}-m & -m & \ddots & \vdots \\ -m & \ddots & \ddots & \ddots & -m \\ \vdots & \ddots & \ddots & \check{k}-m & -m \\ -m & \cdots & -m & -m & \check{k}-m \end{bmatrix}}_{n\times n}. \tag{4}$$

Assumption 1 implies that $\Sigma_{i=1}^{n}\check{x}_i$ is constant for all gap widths $\check{\boldsymbol{u}}$. Then (ignoring the noise $\check{\boldsymbol{n}}$), we have from Eq. 1 that

$$\begin{aligned}\sum_{i=1}^{n}\check{x}_i(t)&=\sum_{i=1}^{n}\left(\check{d}_i+\sum_{j=1}^{n}P_{i,j}\check{u}_j(t-1)\right)\\&=\sum_{i=1}^{n}\check{d}_i+\sum_{j=1}^{n}\left(\sum_{i=1}^{n}P_{i,j}\right)\check{u}_j(t-1)\end{aligned} \tag{5}$$

must be a constant for all $\check{u}_j(t-1)$. This implies that

$$\sum_{i=1}^{n}P_{i,j}=0, \quad \text{for } j=1,2,\ldots,n. \tag{6}$$

By substituting the elements of $\boldsymbol{P}$ from Eq. 4 into the summation (Eq. 6), we find that m must be related to $\check{k}$ by:

$$m=\frac{\check{k}}{n}. \tag{7}$$

Substituting for m in Eq. 4 gives the final form for $\boldsymbol{P}$:

$$\boldsymbol{P}=\frac{\check{k}}{n}\boldsymbol{B}, \tag{8}$$

where

$$\boldsymbol{B}=\underbrace{\begin{bmatrix} n-1 & -1 & -1 & \cdots & -1 \\ -1 & n-1 & -1 & \ddots & \vdots \\ -1 & \ddots & \ddots & \ddots & -1 \\ \vdots & \ddots & \ddots & n-1 & -1 \\ -1 & \cdots & -1 & -1 & n-1 \end{bmatrix}}_{n\times n}. \tag{9}$$

The single model parameter $\check{k}$ does not depend on the number of actuators n.

Dimensionless model

The model is transformed to a dimensionless form for two reasons. First, using a dimensionless model will allow the control parameters to vary little between different plants. Second, the controller is designed to produce a coating of uniform thickness and will *not* be able to change the mean coating thickness. A flow controller which maintains constant flow to the coating die is used to adjust the mean coating thickness. Therefore the nondimensional variable $\boldsymbol{x}$ is chosen to represent coating thickness as a deviation from the mean.

Define $\bar{x}=(1/n)\Sigma_{i=1}^{n}\check{x}_i$ and $\bar{u}$ as the nominal gap width. The nominal gap width should be chosen well within the stable coating region. Define the following dimensionless variables:

$$x_i=\frac{\check{x}_i-\bar{x}}{\bar{x}}, \quad u_i=\frac{\check{u}_i-\bar{u}}{\bar{u}}, \quad d_i=\frac{\check{d}_i-\bar{x}}{\bar{x}}, \quad n_i=\frac{\check{n}_i}{\bar{x}}, \quad k=\frac{\check{k}\bar{u}}{n\bar{x}}. \tag{10}$$

Solve the above expressions for $\check{x}_i$, $\check{u}_i$, $\check{d}_i$, $\check{n}_i$, and $\check{k}$, substitute into Eq. 1, and rearrange to give the dimensionless model:

$$\boldsymbol{x}(t)=k\boldsymbol{B}\boldsymbol{u}(t-1)+\boldsymbol{d}+\boldsymbol{n}(t). \tag{11}$$

Model for control design

The matrix $\boldsymbol{B}$ in Eq. 9 is singular. This is because the coating thicknesses $\boldsymbol{x}$ are not uniquely determined by the gap widths $\boldsymbol{u}$. Any increment in gap width added to all the gap widths u_i does not change the coating thicknesses. However, to keep a stable film, the dimensionless gap widths $\boldsymbol{u}$ must not stray too far from the preferred position of $\boldsymbol{0}$. We augment the model with the additional equation $\Sigma_{i=1}^{n}u_i=0$ to both keep $\boldsymbol{u}$ from straying and to give a unique mapping of the coating thicknesses to the gap widths. This is done as follows:

- Add a component to $\boldsymbol{x}$, $\boldsymbol{d}$, and $\boldsymbol{n}$, and set this component to zero, that is, $x_{n+1}=n_{n+1}=d_{n+1}=0$.
- Add a row of ones to the plant matrix $k\boldsymbol{B}$ to give the new $(n+1)\times n$ plant matrix

$$\boldsymbol{C}=\begin{bmatrix} k\boldsymbol{B} \\ 1 \ldots 1 \end{bmatrix}.$$

This leads to the augmented model

$$\boldsymbol{x}(t)=\boldsymbol{C}\boldsymbol{u}(t-1)+\boldsymbol{d}+\boldsymbol{n}(t). \tag{12}$$

Since the mean value of $\boldsymbol{u}$ is a *free* independent variable (it does not change coating thicknesses), a controller design based on the above model which seeks to minimize $\boldsymbol{x}$ will automatically adjust its control action so that the mean value of $\boldsymbol{u}$ will be exactly zero. Also, the singularity of the original gain matrix $\boldsymbol{B}$ is removed; $\boldsymbol{C}$ has full column rank.

To derive the model predictive controller in the next section, it is convenient to express the model in terms of the changes in the inputs rather than the inputs themselves. For this purpose, we subtract Eq. 12 for $t-1$ from that at t to arrive at

$$\boldsymbol{x}(t)=\boldsymbol{x}(t-1)+\boldsymbol{C}\Delta\boldsymbol{u}(t-1)+\Delta\boldsymbol{n}(t), \tag{13}$$

where

$$\Delta\boldsymbol{u}(t-1)=\boldsymbol{u}(t-1)-\boldsymbol{u}(t-2). \tag{14}$$

The controller calculates the inputs to the plant based on the measured variables. The model for control design is:

$$x(t) = x(t-1) + C\Delta u(t-1). \tag{15}$$

Estimation and Prediction

Recall that our objective for using a model is to predict the effect of changes in gap widths on the coating thicknesses. This will allow us to find the "best" adjustments in gap widths to reject disturbances.

State estimation—filter

The state estimator is most conveniently expressed in the following two-step form (Morari and Lee, 1991; Morari et al., 1992):

Model Prediction:

$$x(t|t-1) = x(t-1|t-1) + C\Delta u(t-1). \tag{16}$$

Correction Based on Measurements:

$$x(t|t) = x(t|t-1) + \gamma[\hat{x}(t) - x(t|t-1)],\ \gamma\epsilon(0,\ 1]. \tag{17}$$

$x(\cdot|t-1)$ denotes the estimate of $x(\cdot)$ based on measurements up to time $t-1$. $\hat{x}(t)$ is the measurement of x at time t. $\gamma\epsilon(0,1]$ is a filter parameter used to filter noise and to obtain robustness to model uncertainty. The larger the measurement noise and model uncertainty, the smaller γ should be chosen.

By substituting Eq. 16 into Eq. 17 we obtain the *state estimator*

$$x(t|t) = (1-\gamma)[x(t-1|t-1) + C\Delta u(t-1)] + \gamma\hat{x}(t), \tag{18}$$

which allows one to compute the current state estimate $x(t|t)$ based on the previous estimate $x(t-1|t-1)$, the previous input move $\Delta u(t-1)$, and the current measurement $\hat{x}(t)$. The state estimator is initialized with $x(0|0) = \hat{x}(0)$.

The state estimator (Eq. 18) suggests that $x(t|t)$ is a filtered version of $\hat{x}$. Indeed, in a noise-free system with the manipulated variables constant, we have

$$x(t|t) = (1-\gamma)x(t-1|t-1) + \gamma\hat{x}(t), \tag{19}$$

which shows that the state estimate $x(t|t)$ is $\hat{x}$ passed through a first-order filter. If the output $\hat{x}$ suddenly changes to a constant value then the state estimate $x(t|t)$ approaches the true value $\hat{x}$ with the filter time constant:

$$\tau = \frac{T_s}{\log\left(\frac{1}{1-\gamma}\right)}, \tag{20}$$

where T_s is the time between sampling instances (Morari and Lee, 1991; Morari et al., 1992).

Prediction

The control algorithm prescribes the gap widths u which reject disturbances in x. In order for the control algorithm to determine the "best" current gap widths there has to be a means for *predicting* the effect of the gap widths on the future coating thicknesses x. The predictor is given by writing Eq. 16 for the next time step $t+1$:

$$x(t+1|t) = x(t|t) + C\Delta u(t) \tag{21}$$

Control

We begin by stating the unconstrained control objective. We derive the unconstrained controller that minimizes the objective. Then we discuss three methods of modifying this controller to handle actuator constraints, in our case constraints in adjacent gap widths.

Unconstrained control algorithm

Performance Criterion. The performance criterion is to minimize the quadratic objective:

$$z = \|x(t+1|t)\|^2, \tag{22}$$

where $\|\cdot\|$ represents the Euclidean norm, $\|x\|^2 = \Sigma_{i=1}^{n} x_i^2$.

Unconstrained Control Problem. We express the control problem as an optimization by combining the objective (Eq. 22) with the predictor (Eq. 21):

$$\min_{\Delta u(t)} \|x(t+1|t)\|^2,$$

$$\text{where } x(t+1|t) = x(t|t) + C\Delta u(t). \tag{23}$$

The least-squares solution to the unconstrained control problem is:

$$\Delta u(t) = -(C^TC)^{-1}C^Tx(t|t). \tag{24}$$

Methods for handling actuator constraints

Excessive stresses in the die constrain adjacent actuator positions. We will consider two ways of specifying these constraints. First, the specification could be that the difference between adjacent actuator positions is limited, that is,

$$|\delta u_i| = |u_{i+1} - u_i| \leq |\delta u|_{\max,} \quad \text{for } i = 1,\ldots,n-1. \tag{25}$$

An additional specification could be that the difference between adjacent actuator positions must be even less when large adjacent gap differences are made in opposite directions. This constraint can be written as:

$$|\delta^2 u_i| = |u_{i+2} - 2u_{i+1} + u_i| \leq |\delta^2 u|_{\max,} \quad \text{for } i = 1,\ldots,n-2. \tag{26}$$

For those plants where $|\delta^2 u|_{\max} \geq 2|\delta u|_{\max}$, the first constraint (Eq. 25) implies the second constraint (Eq. 26), so for these plants the second constraint need not be considered.

Constraint-handling will be needed when the disturbances are sufficiently large and have sharp *spatial* variations across the substrate. When the disturbances are uniform across the substrate, then the control action calculated from the unconstrained control algorithm will be uniform, and constraint-handling is not needed.

Actuator constraints can be handled in three ways: by including additional terms in the objective function, by adding the constraints explicitly to the control algorithm, or by scaling the control actions to be "feasible", that is, to satisfy the constraints. Below we describe each method of handling actuator constraints. We will choose the simplest, yet effective, constraint-handling method for our control problem.

Additional Terms in the Objective Function. Additional terms weighting $|u_{i+1}-u_i|$ and $|u_{i+2}-2u_{i+1}+u_i|$ could be added to the objective function (Eq. 22), that is,

$$z=\|\mathbf{x}(t+1|t)\|^2+\beta_1\sum_{i=1}^{n-1}|u_{i+1}-u_i|^2+\beta_2\sum_{i=1}^{n-2}|u_{i+2}-2u_{i+1}+u_i|^2. \tag{27}$$

The disadvantage of this approach is that the added weighted terms *always* affect the control action. The weights for these terms must be large enough to keep the control action feasible for disturbances which contain sharp spatial variations, but large weights on the control action will substantially slow the control action when the disturbances are uniform across the substrate and the extra terms are not needed.

Explicitly Adding Constraints to the Control Algorithm. The constraints could be added explicitly to the control algorithm. Then the constrained control problem will be the unconstrained control problem (Eq. 23) plus the additional constraints (Eqs. 25 and 26):

$$\min_{\Delta u(t)}\|x(t+1|t)\|^2, \tag{28}$$

such that $x(t+1|t)=x(t|t)+C\Delta u(t)$

$$\begin{aligned}&|\delta u_i|=|u_{i+1}-u_i|\le|\delta u|_{\max}, && \text{for } i=1,\ldots,n-1.\\ &|\delta^2u_i|=|u_{i+2}-2u_{i+1}+u_i|\le|\delta^2u|_{\max}, && \text{for } i=1,\ldots,n-2.\end{aligned} \tag{29}$$

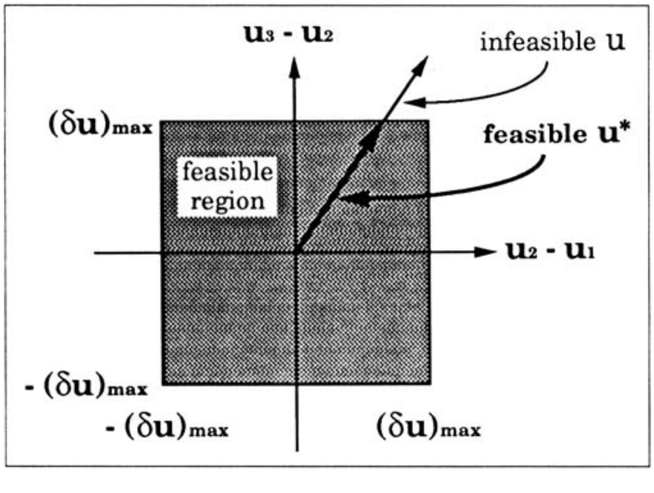

Figure 2. Projection of an infeasible control action to the feasible space.

This is a quadratic programming problem that must be solved at each time step for the optimal actuator movements $\Delta u(t)$. This approach is not as simple to implement and analyze as the third constraint-handling method discussed next.

Scaling Control Actions. Constraints can be handled by projecting any infeasible u given by the unconstrained control law (Eq. 24) to the feasible space. Figure 2 illustrates this idea for the first constraint (Eq. 25) for $n=3$. All feasible control actions u are given by the shaded region. When the unconstrained control law (Eq. 24) suggests an infeasible control action, a feasible control action is found by projecting u to the feasible space. Many projections could be used, but the projection shown (which involves simple scaling of the control action) maintains the direction of the control action, which can be important for multivariable systems (Campo, 1990).

Now consider satisfying the first constraint (Eq. 25) for general n. This is done by scaling the control action u calculated from the unconstrained control law (Eq. 24):

$$u^*(t)=\begin{cases}u(t) & \max_i|\delta u_i(t)|\le|\delta u|_{\max}\\ \dfrac{|\delta u|_{\max}}{\max_i|\delta u_i(t)|}u(t) & \max_i|\delta u_i(t)|>|\delta u|_{\max}.\end{cases} \tag{30}$$

In addition, the control action from the above equation can be scaled to satisfy the second constraint (Eq. 26):

$$u^\dagger(t)=\begin{cases}u^*(t) & \max_i|\delta^2u_i^*(t)|\le|\delta^2u|_{\max}\\ \dfrac{|\delta^2u|_{\max}}{\max_i|\delta^2u_i^*(t)|}u^*(t) & \max_i|\delta^2u_i^*(t)|>|\delta^2\mathrm{u}|_{\max}.\end{cases} \tag{31}$$

$u^\dagger$ satisfies both constraints (Eqs. 25 and 26).

This constraint-handling method is easy to implement and performs exactly as the unconstrained algorithm when constraint handling is not needed. It is shown in Braatz et al. (1991) that, provided the assumptions stated previously hold, the scaling method performs nearly as well as explicitly adding the constraints to the control algorithm.

Constrained Control Algorithm. In summary, the constrained control algorithm is:

- Calculate the estimated state through Eq. 18.
- Calculate the unconstrained control move from Eq. 24.
- Scale the unconstrained control move using Eqs. 30 and 31 to obtain the constrained control move which is implemented. The state estimator for the next step (Eq. 18) will use the *constrained* implemented move from the previous step.

Limits of Performance

We would like to know how well the controller can be expected to reject disturbances in coating thicknesses. This leads us to study the various factors that *limit* the achievable closed-loop performance. Knowledge of how these limitations affect the performance can show us how to modify the plant to improve the uniformity of the coating process. Also, because identification of model parameters is time-consuming and costly, we study how accurate the identification must be to achieve a given level of performance. We would also like to compare the performance of our control algorithm to the best closed-loop performance achievable by any control algorithm. This allows us to convince ourselves that we have indeed designed the best possible controller.

We begin by making the assumptions necessary to achieve perfect one-step rejection of disturbances. This provides a standard to which the various limitations on the closed-loop performance can be compared.

Perfect Control. We are interested in the ability of the controller to reject slow disturbances. Let us study the rejection of a steady-state disturbance and let the control algorithm start at $t=0$. For simplicity of presentation, let the disturbance $\boldsymbol{d}$ have zero-mean and the initial gap widths $\boldsymbol{u}(-1)=0$. If we make the following three assumptions:

1. No actuator constraints
2. No measurement noise
3. Our model is exactly equal to our plant

then it can be shown that the control algorithm with $\gamma=1$ perfectly rejects the steady-state disturbance in one step.

We will drop the assumptions of no actuator constraints, no measurement noise, and no model uncertainty in turn and show how each of these prevent the controller from rejecting the steady-state disturbance in one step. We will also investigate if the plant condition number limits performance.

Constraints on actuator movements

The constraints on the actuator positions will degrade performance only when the control move from the unconstrained algorithm must be scaled to keep the gap widths feasible. It can be shown that in this case the coating thicknesses at the next time $\boldsymbol{x}(1)$ do not equal zero. We will also show below that the coating thicknesses $\boldsymbol{x}$ may never reach zero.

Assume no measurement noise, $\gamma=1$, that the model is perfect, and for simplicity of presentation that $\boldsymbol{d}$ has zero mean and the initial gap widths $\boldsymbol{u}(-1)=0$. Then the measured coating thicknesses at $t=0$ is $\hat{\boldsymbol{x}}(0)=\boldsymbol{x}(0)=\boldsymbol{d}$. The control move for the first step from Eq. 24 is:

$$\boldsymbol{u}(0)=-(\boldsymbol{C}^T\boldsymbol{C})^{-1}\boldsymbol{C}^T\boldsymbol{d}. \tag{32}$$

If the control move from the unconstrained algorithm must be scaled to keep the gap widths feasible, the constrained control move is:

$$\boldsymbol{u}^{\dagger}(0)=-\lambda(\boldsymbol{C}^T\boldsymbol{C})^{-1}\boldsymbol{C}^T\boldsymbol{d}, \tag{33}$$

where $0<\lambda<1$. If the operator implements the control move $\boldsymbol{u}^{\dagger}(0)$ exactly and there is no measurement noise, then applying the control move to the plant (Eq. 12) gives that (after some matrix manipulation):

$$\boldsymbol{x}(1)=(1-\lambda)\boldsymbol{d}. \tag{34}$$

We see that the effect of the disturbance has been diminished by a factor of $1-\lambda$. It can be shown that under the given assumptions, the control move will not change, and the coating thicknesses will continue to be $\boldsymbol{x}(t)=\boldsymbol{x}(1)=(1-\lambda)\boldsymbol{d}$.

The constraints on gap widths prevent the steady-state disturbance from being completely rejected. This is true regardless of the control algorithm used.

Plant Modifications to Improve Performance. The gap widths are constrained to prevent high stresses in the die. A die can be designed to have weaker constraints on its die gap widths by placing the bolts further apart, by making the die lip thinner, or by making the die out of a more flexible metal. Putting the die bolts too far apart leads to strips of uncontrolled coating thickness between the die bolts. Machining a die to tight tolerances becomes increasingly difficult as the die metal becomes thinner or more flexible.

Measurement noise

Measurement noise always limits performance. A noise filter is used to diminish the effects of noise. Because increased noise filtering also slows the controller response time, there is a trade-off between improved coating uniformity and slower response times. We now define a measure of coating uniformity and study this trade-off in more detail.

Consider the closed-loop system with a perfect model without disturbances and only measurement noise. For a stabilizing controller, the expected value for the estimated state $x(t|t)$ is zero. The estimated state will not exactly equal zero because the controller will treat the measurement noise as a disturbance and will try to reject it. Thus the estimated state will have some variance depending on the size of the noise. The variance of the estimated state $x(t|t)$ is an appropriate measure of the uniformity of the coating. For simplicity of presentation, assume a perfect model in which the noise at each gap position is equal—dropping these assumptions only slightly affects the following. Then it can be shown that:

$$\text{Variance }(x_i)=\frac{\gamma}{2-\gamma}\text{ Variance }(n_i)\text{ for } i=1,\ldots,n. \tag{35}$$

A measure of the controller's speed of response is the filter time constant plus 1, that is, $\tau+1$ (The '1' accounts for the delay through the plant).

Both Variance (x_i) and τ (through Eq. 20) are functions of the noise filter parameter γ. Figure 3 compares the controller response time vs. the ratio of the variance of the state estimate to the measurement noise variance for different values of γ.

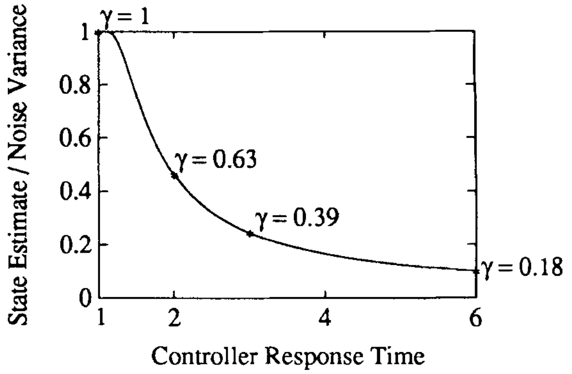

Figure 3. Relationship between coating uniformity and controller response time.

A small amount of filtering ($\gamma \to 1$) corresponds to fast response times, but poor coating uniformity. A large amount of filtering corresponds to good coating uniformity, but with slow response times.

Plant Modifications to Improve Performance. Ways to decrease the sensor noise should be investigated. The cables to the sensor should be shielded adequately to keep the sensor noise as small as possible. The effect of air currents can be diminished by decreasing the distance between the sensor and the coated substrate. The vibration of the substrate and the sensor should be minimized. Of course, an accurate sensor reading requires a stable film.

Model uncertainty

Model Uncertainty. It is the difference between the model and the plant. The error between the true behavior of the physical process and that predicted by the model can significantly affect the ability of the control system to perform adequately. Controllers that are insensitive to model uncertainty are said to be *robust*. Below we quantify the effect of uncertainty. More specifically, we show that the control algorithm proposed in this article is robust to gain uncertainty. Also, we will analyze the robustness as a function of the filter parameter γ to determine the effect of the noise filter on robustness.

Uncertainty in Gain Matrix. The closed-loop stability can be analyzed from the state-space equation for the closed-loop system. A system will be considered stable when the effect of small disturbances remains small. A system is considered unstable when the effect of small disturbances grows until the constraints (Eqs. 25 and 26) are reached. The effect of disturbances will never grow unbounded because the constraints (Eqs. 25 and 26), and $\Sigma_{i=1}^{n} u_i = 0$ hold, which bound the magnitude of the control action.

Let the measurement be described in terms of the real plant:

$$\hat{x}(t) = C_r\, u(t-1) + v_r(t). \tag{36}$$

No assumptions are made on the unmeasured inputs v_r.

Define Γ by

$$\Gamma = -(C^T C)^{-1} C^T. \tag{37}$$

Then the control law (Eq. 24) is given by:

$$u(t) = u(t-1) + \Gamma x(t|t). \tag{38}$$

Substitute $x(t|t-1)$, $\hat{x}(t)$, and $u(t-2)$ from Eqs. 16, 36 and 38 into Eq. 18 and rearrange to give:

$$x(t|t) = (1-\gamma)(I + C\Gamma)x(t-1|t-1) + \gamma C_r\, u(t-1) + \gamma v_r(t). \tag{39}$$

Substitute $x(t|t)$ from Eq. 39 into Eq. 38 to give:

$$u(t) = (1-\gamma)\Gamma(I + C\,\Gamma)x(t-1|t-1) + (I + \gamma\Gamma C_r)u(t-1) + \gamma\Gamma v_r(t). \tag{40}$$

Let $u(t)$ be a state, then Eqs. 39 and 40 give the state-space equation that defines the closed-loop system,

$$\begin{bmatrix} x(t|t) \\ u(t) \end{bmatrix} = \begin{bmatrix} (1-\gamma)(I + C\Gamma) & \gamma C_r \\ (1-\gamma)\Gamma(I + C\Gamma) & I + \gamma\Gamma C_r \end{bmatrix} \times \begin{bmatrix} x(t-1|t-1) \\ u(t-1) \end{bmatrix} + \begin{bmatrix} \gamma \\ \gamma\Gamma \end{bmatrix} v_r(t). \tag{41}$$

For a discrete time system, we have closed-loop stability if and only if the eigenvalues of

$$A = \begin{bmatrix} (1-\gamma)(I + C\Gamma) & \gamma C_r \\ (1-\gamma)\Gamma(I + C\Gamma) & I + \gamma\Gamma C_r \end{bmatrix} \tag{42}$$

are inside the unit circle. More specifically, the effect of disturbances will decay to zero if the spectral radius of A is less than one, and the effect of small disturbances will grow until the constraints are met when the spectral radius of A is greater than one (Åström and Wittenmark, 1984).

Uncertainty in Gain. This section considers uncertainty in the gain; interaction uncertainty for the Avery/Dennison pilot plant will be considered later. The real plant gain will be denoted as k_r and the augmented real plant is:

$$C_r = \begin{bmatrix} k_r B \\ 1 \cdots 1 \end{bmatrix}. \tag{43}$$

Recall that k is the gain and C is the gain matrix for the model.

By calculating the eigenvalues of A in Eq. 42 we determine which values of the ratio $K = k/k_r$ give a stable closed-loop system for each value of filter parameter γ (see Figure 4). If the gain of the real plant is not underestimated by more than a factor of two ($K > 1/2$), then the closed-loop system is stable. For increased filtering (smaller γ), the model gain k need not be as accurate. In other words, increased filtering adds robustness to gain uncertainty. It can be shown that the stability boundary in Figure 4 is the straight line given by $k = (1/2)\gamma k_r$.

The plant gain need not be known accurately for the closed-loop system to be stable. Uncertainty in the plant gain will lead only to slower rejection of disturbances. Since we need approximate only a plant gain to design the controller, detailed identification runs are unnecessary for controller design. Any

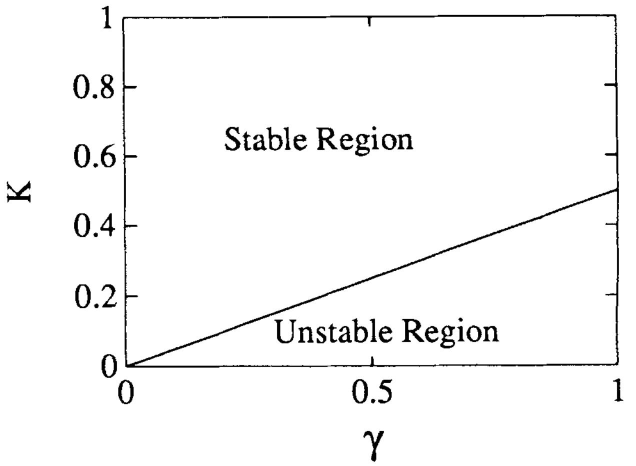

Figure 4. Closed-loop stability as a function of γ and $K = k/k_r$, no interaction uncertainty.

reasonable estimate will do. This makes it easier to apply the control algorithm to new cross-directional systems when $\check{k}$ does not change much between systems.

Plant condition number

It is well-known that high condition number plants (called *ill-conditioned*) can be difficult to control (Morari and Doyle, 1986; Skogestad and Morari, 1987; Skogestad et al., 1988). By the condition number we mean:

$$\kappa(C) \equiv \frac{\overline{\sigma}(C)}{\underline{\sigma}(C)}, \tag{44}$$

where $\overline{\sigma}$ and $\underline{\sigma}$ denote the maximum and minimum singular values of the plant:

$$\overline{\sigma}(C) = \max_{u \neq 0} \frac{\|Cu\|_2}{\|u\|_2}, \quad \underline{\sigma}(C) = \min_{u \neq 0} \frac{\|Cu\|_2}{\|u\|_2}. \tag{45}$$

A plant with a high condition number is characterized by strong directionality because inputs in directions corresponding to high plant gains are strongly amplified by the plant, while inputs in directions corresponding to low plant gains are not. Thus, ill-conditioned plants may be sensitive to actuator uncertainty (Skogestad et al., 1988).

Recall that

$$C = \begin{bmatrix} kB \\ 1 \cdots 1 \end{bmatrix}.$$

The last row of C was augmented to the plant matrix kB to keep u from straying from zero. The elements of the last row of C need not be 1's—the last row can be any constant multiplied by a row of 1's. Because the controllability of the process is not dependent on what scalar is used in the last row of C, a true measure of the controllability of the process must be independent of this scalar. A "true" measure of the controllability of the plant can be defined as:

$$\kappa^*(C) \equiv \inf_s \kappa\left(\begin{bmatrix} kB \\ s \cdots s \end{bmatrix}\right). \tag{46}$$

It can be proven using the theory of circulant matrices (Davis 1979; Hovd, 1992) that $\kappa^*(C) = 1$ for all n (the s that minimizes the condition number in Eq. 46 is $s = \sqrt{n}$). This means that ill-conditioning is not a serious problem for cross-directional processes of the type studied here.

Application to Avery/Dennison Pilot Coater

The control algorithm is applied to a pilot-plant coater at Avery/Dennison Research Center. (All figures and data are given in terms of dimensionless variables for proprietary reasons.) (see Figure 1). Typical ranges of physical parameters for such coaters are given in Table 1.

First, the model is identified and the model assumptions are justified based on input-output data. Then, the effect of interaction uncertainty on the stability of the closed-loop system was investigated using the model fit to the pilot-plant data.

Table 1. Typical Ranges of Physical Parameters for Adhesive Coaters

Die Width	0.35–2.5 m
Die Bolt Spacing	30–60 mm
Coating Thickness	10–60 μm
Coating Weight	15–50 g/m^2
Substrate Speed	0.5–6 m/s

This was done to ensure that uncertainty in the interactions (that is, deviations from the structure implied by Eq. 4) would not cause the controller to perform poorly. We then demonstrate that the controller can be effectively tuned on-line. We conclude the section with an experimental closed-loop test of the controller.

Identification

For the pilot plant, the number of actuators $n = 12$. k was fitted by least squares from 50 input-output data sets. In Figure 5 the predicted coating thicknesses are compared with experimental data for a typical input.

To test the assumptions used to develop the form of the gain matrix P described earlier, we fitted the entire 12×12 gain matrix in Eq. 1 to estimate a total of 144 parameters—we denote this matrix by P_{144}. As shown in Figure 5, this model gives little improvement over the gain matrix P satisfying the assumptions, so the assumptions on P are valid.

The die had been designed to give a small interaction between nearest-neighbor positions. Assumption 3 would not have been justified if the spacing between the actuators had been much smaller.

Robustness to interaction uncertainty

The effect of interaction uncertainty on the stability of the closed-loop system was investigated using the model fit to the pilot-plant data. This was done to ensure that uncertainty in the interactions would not cause the controller to perform poorly. The procedure described earlier was used, but with

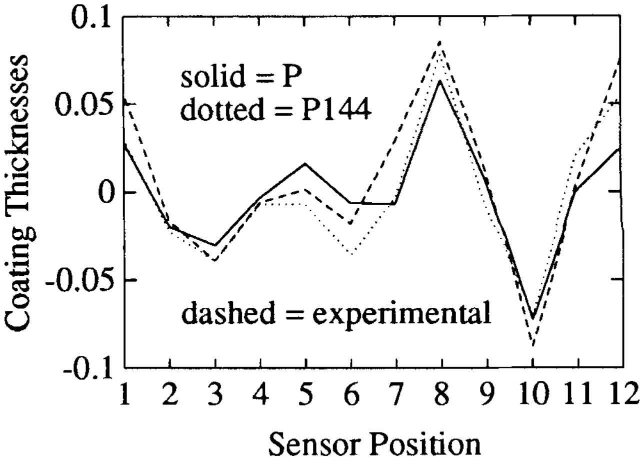

Figure 5. Comparison of coating thicknesses predicted by P and P_{144} with experimental data.

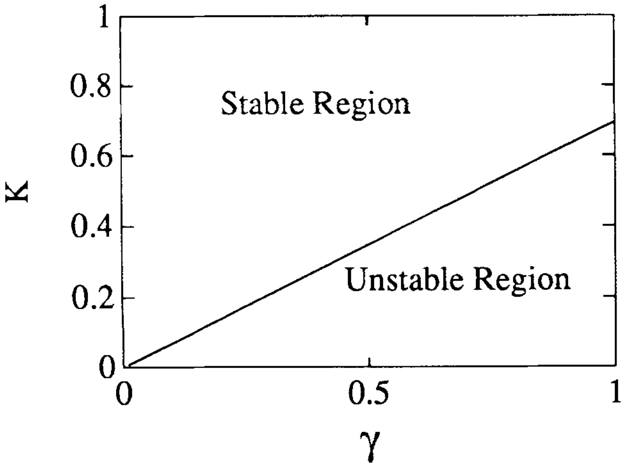

Figure 6. Closed-loop stability as a function of γ and $K=k/k_r$.

Interaction uncertainty was included through the use of P_{144}.

$$C_r=\begin{pmatrix} P_{144} \\ 1\cdots 1 \end{pmatrix} \text{ for the real plant}$$

and

$$C=\begin{pmatrix} kB \\ 1\cdots 1 \end{pmatrix} \text{ for the model.}$$

Figure 6 shows the stable region as a function of the normalized model gain $K=k/k_r$ where k_r denotes the best fit gain in C. As in Figure 4, the boundary between the stable and unstable regions is a straight line, but the slope in Figure 6 is steeper. Introducing interaction uncertainty decreases the stable region, but an accurate estimate of k is still not required. This will be experimentally verified below.

Experimental closed-loop control

The main purpose of the experiments was to verify that detailed identification of k is not required for the resulting

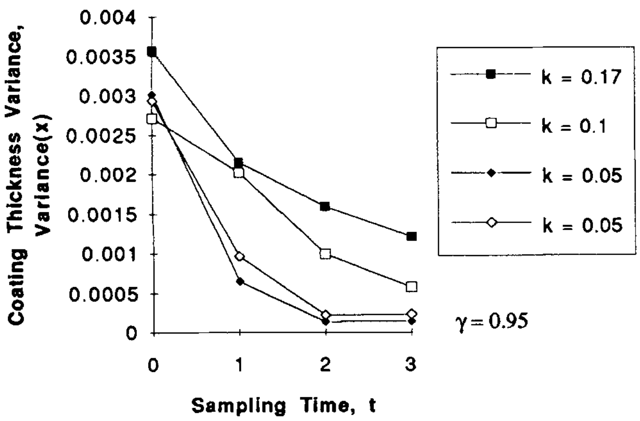

Figure 7. Comparison of coating thickness variances.

The control actions calculated using $k=0.025$ were excessively large and were not implemented.

controller to give good performance. This is important because gathering detailed input-output data is expensive.

All the die gaps were set equal to their nominal value. Because of imperfections in the die and roller and inaccuracy in the die gap settings, this gives nonuniform coating thicknesses. The goal of the controller is to make the coating thicknesses uniform. This disturbance is small enough that constraint-handling was not needed. Because the number of experiments was limited, we decided to perform all experiments with a fixed γ near one. As discussed earlier, in plant operation γ would be chosen to trade off the closed-loop speed of response with the variance of the coating thicknesses.

There were two major differences between the coater used for the identification experiments and the coater used for the closed-loop experiments. First, the measurement noise was smaller for the second coater. Second, the coaters had different dies, so the responses with the two dies are expected to be different. A comparison of die designs showed that the interactions are negligible for both dies, but the steady-state gains k are expected to be substantially different. Because experiments are costly, out strategy was to avoid re-identifying k from open-loop experiments but to perform closed-loop experiments instead for a few values of k and choose the one that gives good control—effectively determining the optimal k through on-line tuning.

Figure 7 shows the variance of the coating thicknesses for $k=0.17$, 0.1, and 0.05. Since γ was chosen near 1 and the interactions were negligible, we expect a fast response when the model steady state gain k is close to the true gain. Because the gain $k=0.17$ identified for the previous die gave slow response, the controller gain is too small. This implies that the steady-state gain for the model is too large. The response for $k=0.1$ also gave sluggish response. Therefore we tried a smaller k. For $k=0.05$, the disturbance was rejected in two sampling times.

If we had perfect control and $\gamma=1$, the disturbance would be rejected in one sampling time. If the assumptions of perfect control described previously were satisfied with $\gamma=0.95$, then the closed-loop time constant would be

$$\tau+1=\frac{1}{\log[1/(1-0.95)]}+1\approx 4/3>1.$$

Since we do not satisfy all the assumptions of perfect control, we cannot expect the disturbance to be rejected in less than two sampling times, that is, $k=0.05$ gives the best achievable performance. We see that k needed to design the controller was determined from only three closed-loop experiments.

From Figure 6 we expect that using k much less than 0.05 would give poor performance. This agrees with experiment—the control actions calculated using $k=0.025$ were excessively large and were not implemented.

Figure 8 shows the closed-loop response for $k=0.05$. The disturbance was not completely rejected by the controller because of measurement noise and stiction-like effects in the die gaps.

The purpose of the next closed-loop experiment was to test the closed-loop performance with the controller designed above ($k=0.05$, $\gamma=0.95$). Figure 9 shows the closed-loop response (the variance of the coating thicknesses) with the designed controller to two types of disturbances. The first disturbance

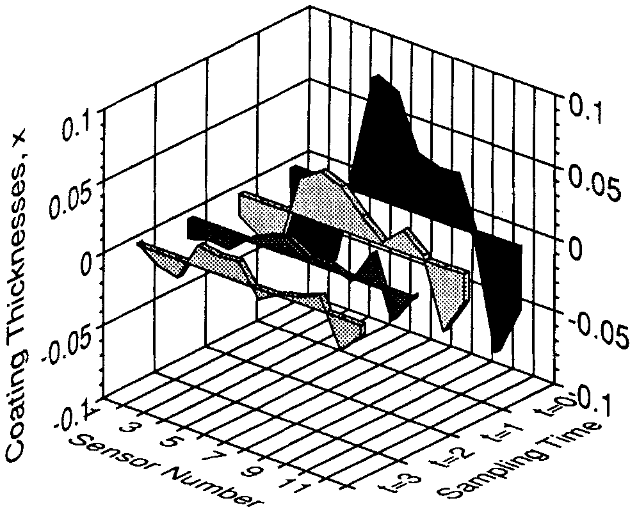

Figure 8. Closed-loop response for $k=0.05$.

was caused by a roller that had a larger radius for the intermediate sensor positions than for the edge positions—this disturbance was rejected within two sampling times as shown in Figure 9. The second disturbance was caused by ramping the roller speed and liquid flow rates (in a constant ratio) to double their values between the fourth and fifth sampling instances. The nominal gap width was kept at a constant value. We see from Figure 9 that changing the roller speed and liquid flow rates in a constant ratio does not substantially affect the variance of the coating thicknesses.

Conclusions

A model-predictive control algorithm was presented which rejects slow disturbances in coating thicknesses for a class of industrial coating processes. The control algorithm has one tuning parameter γ, which trades off robustness to model error and insensitivity to measurement noise with speed of response.

Several constraint-handling methods were discussed. The simplest, yet effective, constraint-handling method involved scaling the control action by a scalar which was just large enough to make the control action feasible.

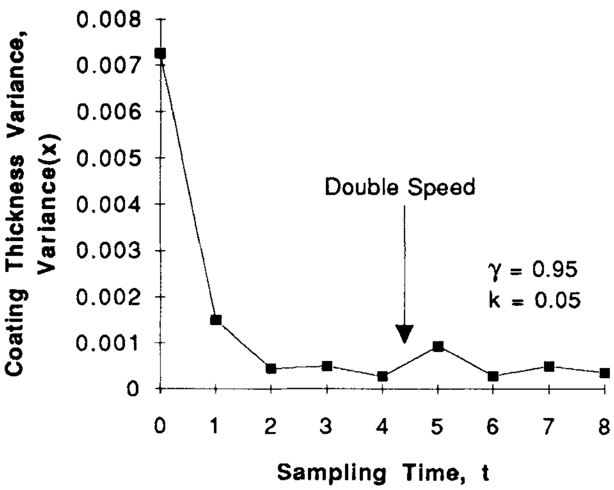

Figure 9. Closed-loop response for two disturbances.

Actuator constraints, measurement noise, model uncertainty, and the plant condition number are investigated to determine which of these limit the achievable closed-loop performance. Knowledge of how these limitations affect the performance suggests how to modify the plant and the controller tuning parameter γ to improve the uniformity of the coating process. We showed quantitatively how varying γ determines the trade-off between the estimated final coating uniformity (the variance of the state estimate) and the controller response time. Little filtering ($\gamma \rightarrow 1$) leads to fast response times, but poor coating uniformity. Much filtering leads to good coating uniformity, but with slow response times. We also showed that as long as the interactions were negligible, the closed-loop response was insensitive to a poor gain estimate used to design the controller. This allows the controller to be tuned *on-line*. We determined that the robustness of the controller improved with increased noise filtering and that the plant condition number was not a limitation on closed-loop performance.

The model-predictive controller was applied to a pilot-plant coating process at the Avery/Dennison Research Center in Pasadena. The assumptions described earlier were shown to be valid, so the theory developed throughout this article could be applied directly. The plant gain was determined on-line, and the resulting controller rejected disturbances within two sampling times. Figure 3 shows that the proposed controller can reduce variance in coating thickness by as much as 80% compared to what is possible by manual control or simple control schemes.

The control algorithm developed here can be applied to processes other than coating, for example, to the control of paper machines (Laughlin, 1988), as long as the previous assumptions are valid. The most restrictive assumption regarding the form of the plant matrix $\boldsymbol{P}$ is that the only interactions are due to the constant flow assumption. Additional interactions make the analysis and control much more complex. When handling constraints in the general case, noticeable improvement in performance can be obtained by explicitly adding constraints to the control algorithm, instead of the simple method of scaling the control action which was acceptable here. Significant interaction uncertainty makes plots such as Figures 4 and 6 more difficult to determine and less useful. The plant condition number can become a serious limitation on closed-loop performance. Laughlin (1988) gives examples of plants with only nearest-neighbor interactions for which the condition numbers are infinity—this implies that the systems are uncontrollable. He also presents a general method for handling gain, interaction, and dynamic uncertainties.

On-line tuning becomes difficult when there are interactions—both because the controller depends on the multiple model parameters and because the closed-loop response can be extremely sensitive to poor estimates of the model parameters. When the plant condition number is large, an inexact estimate of the interactions can give an unstable closed-loop system (Skogestad et al., 1988).

This article shows that there are strong advantages to spacing the actuators far enough apart to keep the interactions minimal. The actuators must be spaced close enough together to prevent strips of uncontrolled coating thickness between the actuators. This is how the Avery/Dennison pilot plant was designed.

Acknowledgment

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Notation

B = dimensionless plant gain matrix
C = augmented plant gain matrix
d = dimensionless steady-state disturbance
I = $n \times n$ identity matrix
k = dimensionless plant gain
K = ratio of model plant gain to real plant gain
M = gain matrix due to constant flow assumption
n = number of actuators
$\mathbf{n}$ = dimensionless measurement noise
P = dimensional plant gain matrix
T_s = sampling time
x = dimensionless vector of coating thickness
$\bar{x}$ = average coating thickness
$\hat{x}$ = dimensionless vector of measured coating thicknesses
u = dimensionless vector of gap widths
$\bar{u}$ = nominal gap widths

Greek letters

β_i = weighting parameters
δ = difference in actuator positions
Δ = difference in sampling times
γ = noise filter parameter
Γ = control law matrix
κ = condition number
λ = scaling parameter
$\underline{\sigma}$ = minimum singular value
$\bar{\sigma}$ = maximum singular value
τ = filter time constant

Subscripts and superscripts

$\check{}$ = dimensional
r = real

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