Controllability of Processes with Large Singular Values

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The singular-value decomposition has been used to analyze the robustness of closed-loop systems and to determine whether any controllability problems can be expected. Past controllability conditions have been derived in terms of the condition number and the minimum singular value of the process and the condition number of the controller transfer function matrix, in terms of potential sensitivities of the closed-loop system to model uncertainties or problems with saturation of the manipulated variables. This paper considers processes with a large maximum singular value. It is shown that the closed-loop control of such processes can result in poor transient performance as a result of valve accuracy considerations, *even if the condition number is small and the minimum singular value is large*, which would indicate no performance limitations according to existing controllability criteria. Further, processes with large singular values can be prone to sensor saturation. This indicates that the magnitude of all of the singular values should be considered when assessing the controllability of a process. A new interaction tool based on output correlation is introduced to help select measurements and manipulated variables that have a good range of singular values for practical application. The approach proposed is illustrated on two simple examples and on the Tennessee Eastman process.

Introduction

In overcoming the effects of process disturbances or in achieving desired setpoints, a process control system implicitly or explicitly solves an algebraic problem that depends on the process gain matrix. The control system determines the required manipulated inputs to bring the controlled measurements to their desired steady states. Singular-value decomposition (SVD)[1] can be used to assess just how well this process control algebraic problem is posed and whether any sensitivity problems can be expected when it is solved. In this paper the use of SVD to analyze a process gain matrix is discussed. Without loss in generality, it is assumed that the desired final control system is square, with the number of manipulated and controlled variables being equal, and that integral action is used on all measurements. To use SVD for analyzing a process gain matrix, the matrix must be scaled so that it reflects the actual measurement devices and valves that are used in the plant under study. The particular focus of this paper involves the case where the process gain matrix has a large maximum singular value, with its minimum singular value being either large or small.

Early application of SVD to process control systems was carried out by researchers at the University of Tennessee.[2,3] Lau et al.[4] used SVD to design multivariable control systems. Grosdidier et al.[5] established a quantitative relationship between the condition number of a process gain matrix and its relative gain,[6] and this relationship was extended by Nett and Manousiouthakis.[7] Skogestad and Morari[8] derived conditions for robust stability that involved the condition number. A number of useful results on applying SVD to process control problems are collected by Skogestad and Postlethwaite.[9] This includes the result that saturation of the manipulated variable is a potential problem if the minimum singular value of the process gain matrix is less than 1. They also show that the multivariable effects of input uncertainties are small for processes with small condition numbers. Skogestad and Postlethwaite[9] do present an example of a process with a large condition number where the minimum singular value is greater than 1, but their discussion in this case is more speculative compared to the case where the minimum singular value is less than 1.

There are many common confusions that exist in the literature involving the use of the condition number to analyze processes. For example, it is widely believed that a large condition number of the process always indicates sensitivity to model uncertainties, but this is not true in general, as discussed in detail by Skogestad and Postlethwaite.[9] Another common belief is that a large condition number indicates problems with actuator saturation, but this is not true if the condition number is large and the minimum singular value is large. A much more direct requirement concerning actuator saturation is written in terms of the minimum singular value being small. Hence, it is much more appropriate, and accurate, to use the minimum singular value as the metric for ill-conditioning in terms of actuator saturation[9−11] rather than the commonly used condition number. Using the condition number for processes with a large maximum singular value can confuse the fact that the underlying performance limitation is more directly described in terms of the singular values. For example, a single-input single-output pro-

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cess with a very large gain has a condition number of 1 but has a very large singular value (which is equal to its gain), indicating that the control system may be very sensitive to valve inaccuracies.[12] Rios-Patron and Braatz[12] point out that the large gain is the cause of oscillations commonly observed in pH processes. Hence, the magnitude of singular values is the most accurate and direct way of analyzing a process, and the condition number is an indirect and imprecise way of doing so.

The approach focuses on steady-state conditions, but it is straightforward to extend to the frequency domain. A key result is that processes with large maximum singular values either are prone to sensor saturation or can exhibit transient oscillations because of valve accuracy problems. This motivates the idea that a limit on the maximum singular value should be employed in developing control architectures. A correlation measure is proposed for selecting actuators and sensors so that the process gain matrix has a maximum singular value less than this limit and also a minimum singular value greater than 1. The methods discussed are first applied to two small examples and then to the Tennessee Eastman process.[13] After an introduction to the algebra of process control problems and to SVD, its application in process control is discussed.

Preliminary Considerations: Algebra and Process Control

The goals of the process control system are to suppress disturbances and follow changes in the setpoints. It is assumed that the process control system under consideration is square, that it has n manipulated variables and n measurements to control, and that integral control is used on all measurements. To reject a step disturbance d, this control system will implicitly or explicitly solve the following algebraic problem:

$$y = \mathbf{K}u + d = 0 \tag{1}$$

In eq 1, the measurements, y, and manipulated variables, u, are deviation variables. The setpoint for y is the origin. Assume that the variables u and y are scaled to the range ± 1. One approach to scaling is to use the instrument and valve ranges. For example, a temperature device, y_{T}, with a span of 100 °C could be scaled $(y_{\mathrm{T}} - y_{\mathrm{Tss}})/100$, where y_{Tss} is the steady-state reading. A valve that ranges from full open (100%) to closed (0%) would be scaled as $(u - u_{\mathrm{ss}})/100$. An alternative approach to scaling involves specifying a desired range for a measurement, e.g., temperature within ± 10 °C, and a desired range in which a valve can move, e.g., $\pm 30\%$. If there is an asymmetric difference in how far an actuator can move before saturation, e.g., the actuator is 95% open, then the smaller of the two differences can be used in scaling. The instrument and valve ranges are used for scaling in the following, but the results apply equally well to the approach where the desired range is used. For scaling that involves actual valve and measurement ranges, once a high or low limit is reached (± 1), the valve or measurement device no longer functions and control is lost. For scaling that involves a desired range, reaching a limit means that this range has been violated. If integral control is used, the gain matrix $\mathbf{K}$ is invertible, and the valve constraints are not active, then there exists a controller that brings the measured variables back to the origin. The control system implicitly or explicitly solves eq 1 to determine the inputs that eliminate the effect of the disturbance d as

$$u = -\mathbf{K}^{-1}d \tag{2}$$

For changing setpoints, the control system solves the following algebraic problem:

$$y^{\mathrm{SP}} = \mathbf{K}u \tag{3}$$

with u given by

$$u = \mathbf{K}^{-1}y^{\mathrm{SP}} \tag{4}$$

Equations 2 and 4 demonstrate that the control system inverts the process gain matrix in bringing the process to the desired steady state. Thus, the control system solves an algebraic problem during its operation. If the problem is not well posed, i.e., $\mathbf{K}^{-1}$ cannot be accurately determined, then problems can result with the operation of the control system in the field. It is important to note that process control systems ultimately rely on analogue devices such as transducers and valves and these devices have limited accuracy. As a result, implementing the inverse of $\mathbf{K}$ in a practical control system can be more difficult than calculating $\mathbf{K}^{-1}$ on a computer. SVD can be used to determine if problems may arise when either eq 2 or eq 4 is solved.

SVD of a matrix, $\mathbf{K}$, is given by[1]

$$\mathbf{K} = \mathbf{U}\Sigma\mathbf{V}^{\mathrm{T}} \tag{5}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{\Sigma}$ is a matrix whose diagonal elements are the singular values, σ_i, which are all greater than or equal to 0 and are arranged in decreasing magnitude. In process control applications, singular values of zero typically arise from integrating variables such as liquid levels. [See ref 18 for an example of a process where a zero singular value does not arise from integrating variables.] For these variables, their rate of change can be used in the gain matrix, as discussed below in the Tennessee Eastman application. The off-diagonal elements of $\mathbf{\Sigma}$ are zero. The condition number, CN, of $\mathbf{K}$ is given as the ratio of σ_1 to σ_n as

$$\mathrm{CN} = \sigma_1/\sigma_n \tag{6}$$

Processes with large CNs are considered in the following section.

Control Systems with Large CNs

A large CN can result from a small σ_n, a large σ_1, or both.

Case 1: Small σ_n. This case is discussed extensively by Skogestad and Postlethwaite,[9] and their results are paraphrased below. Assume that a step change in y^{SP} occurs and the control system determines the manipulated variables u as in eq 4. In controllability analysis, it is common to measure the magnitude of the manipulated variables u and the outputs y in terms of the Euclidean norm. [See ref 19 for results where other norms are used.] Further assume that setpoint y^{SP} changes can cover the entire span of outputs y. For this second assumption, y^{SP} is scaled to have a norm of less than 1. The inverse of $\mathbf{K}$ is given by

$$\mathbf{K}^{-1} = \mathbf{V}\Sigma^{-1}\mathbf{U}^{\mathrm{T}} \tag{7}$$

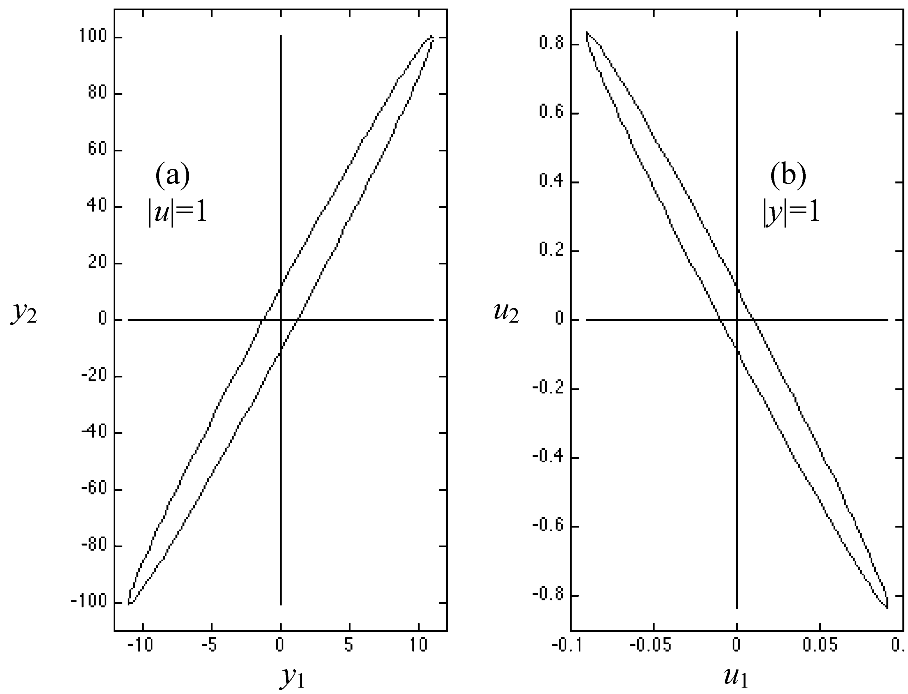

Figure 1. Range of measured variables with the norm of the manipulated variables bounded by 1 and the range of manipulated variables with the norm of the measured variables bounded by 1.

The setpoint change that causes the largest change in u is parallel to u_n, the last column of **U**. This setpoint acts through the inverse of the smallest singular value, $1/\sigma_n$. For a setpoint change of unity magnitude, $y^{SP} = u_n$, the change in u can be calculated by substituting eq 7 into eq 4 to give

$$u = (1/\sigma_n)v_n \tag{8}$$

where v_n is the last column of **V**. For a unitary matrix, each of its column vectors has a magnitude of 1. Thus, $|v_n|_2 = 1$, and u_n constitutes the unity magnitude setpoint change that requires the largest change in u. A sufficient condition to avoid actuator saturation ($|u|_2 \geq 1$) is[9]

$$\sigma_n > 1 \tag{9}$$

If eq 9 is satisfied, then actuator saturation is not a problem. It should be noted that eq 9 involves σ_n and not CN. Also, eq 9 can be conservative because it is calculated based on a worst-case unity magnitude setpoint change. Having a process with $\sigma_n > 1$ is desirable because then actuator saturation is not a problem at steady state whenever a unity magnitude setpoint change is made. Whether actuator saturation actually occurs when $\sigma_n < 1$ depends on the actual setpoint changes encountered. A similar analysis can be used for disturbances. In the case of disturbances, they need to be scaled using the same scaling factors as those for the measurements. If a control system only has to reject disturbances and setpoint changes are not made, then depending on the nature of the disturbances, it may be possible to have a workable control system for which $\sigma_n < 1$.

Case 2: Large σ_1. Another question concerns whether any controllability problems can be anticipated when σ_1 is large. Consider a process whose desired outputs satisfy the constraint $|y|_2 \leq 1$. An orthonormal basis for the space of input u vectors can be chosen as the columns of **V**. Then the maximum allowable magnitude of u along each of the basis directions, α_i, can be calculated to keep $|y|_2 \leq 1$ by using eq 5 to give

$$|y|_2 = |K\alpha_i v_i|_2 = \sigma_i \alpha_i |u_i|_2 \tag{10}$$

Because $|u_i|_2 = 1$, the maximum value of α_i occurs when $|y|_2 = 1$, and it is given by $\alpha_{i,\max} = 1/\sigma_i$. If σ_1 is large and $\sigma_n \geq 1$, then the magnitude of u in some directions, particularly v_1, will be severely limited. To illustrate this result, the following scaled gain matrix is considered:

$$\mathbf{K} = \begin{bmatrix} 11 & 0 \\ 100 & 11 \end{bmatrix} \tag{11}$$

For this plant, $\sigma_1 = 101.2$ and $\sigma_n = 1.196$, indicating that actuator saturation is not a problem. However, this process will have a problem with large output values if the full range of u is used. Figure 1a shows a plot of y_2 versus y_1 for the case where u takes on values from a unit circle, i.e., $|u|_2 = 1$. As Figure 1a shows, the values of y_1 and y_2 are much larger than ± 1, and the norm of y is much greater than 1. If the outputs are to be kept within their spans, then u_1 and u_2 must be limited. To keep $|y|_2 \leq 1$, u_1 and u_2 must remain within the ellipse shown in Figure 1b. Values for u_1 must be restricted to a small fraction of its range, namely, within ± 0.091, when $u_2 = 0$. Valve accuracy is typically specified as a percent of full span, e.g., $\pm 0.5\%$. If a valve is restricted to operate in a smaller range, then its relative accuracy decreases. Thus, a span of ± 0.091 would correspond to a 10-fold decrease in valve accuracy.

Limiting a control valve to a fraction of its range can result in poor control in practice. Consider an analogy from driving an automobile. With an extremely sensitive steering wheel, a small turn on the steering wheel would result in a sharp turn in the automobile. Driving such an automobile would require very cautious manipulations of the steering wheel because it would be very easy to turn the vehicle too far in one direction and then have

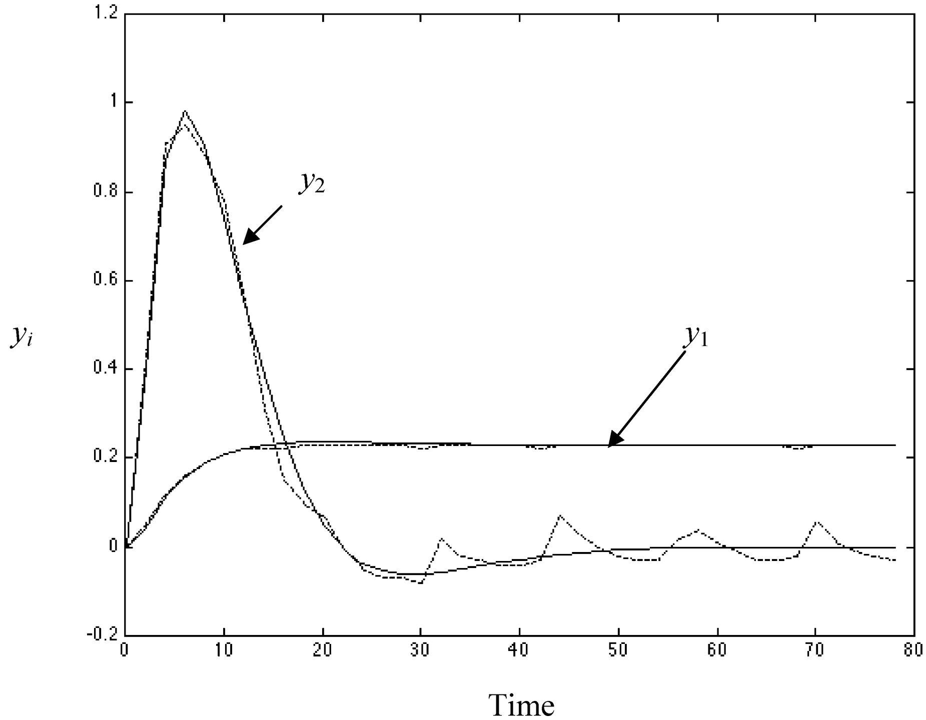

Figure 2. Closed-loop responses with perfect valves (solid lines) and inaccurate valves (dashed lines).

to turn back for compensation. No doubt such an automobile could only be driven at a slow speed, or a continual back and forth motion of the steering wheel would occur. The same type of problem arises for the process described by eq 11. To illustrate this point, a particular first order with dead time process with the following dynamic characteristics is simulated:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{11e^{-s}}{10s+1} & 0 \\ \dfrac{100e^{-s}}{10s+1} & \dfrac{11e^{-s}}{10s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \qquad (12)$$

Two proportional−integral feedback controllers with the following characteristics are used:

$$\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} K_{C1}[1 + 1/(T_{R1}s)] & 0 \\ 0 & K_{C2}[1 + 1/(T_{R2}s)] \end{bmatrix} \begin{bmatrix} \epsilon_1(s) \\ \epsilon_2(s) \end{bmatrix} \qquad (13)$$

where ϵ_i is $y_i^{SP} - y_i$. Identical values of $K_{Ci} = 0.154$ and $T_{Ri} = 7.7$ are used in each loop. If a step change of 0.23 is introduced in the setpoint of y_1, the results shown in Figure 2 are obtained. The size of the setpoint change was chosen so that the output constraint would be satisfied. The solid curves give the responses of y_1 and y_2 when valves with perfect resolution are used. These results are typical of control systems simulated using computers that carry a large number of significant figures. In practice, valve friction and hysteresis would limit the accuracy with which a valve signal could be manipulated.

To illustrate what can happen when σ_1 is large and $\sigma_n \geq 1$, the following simple approach is taken to simulate valve accuracy. It is assumed that a valve signal can only be resolved to a value of ± 0.005, or $\pm 0.25\%$ of the range of each valve, which is equal to 2 (-1 to $+1$). Once values of y_1 and y_2 are measured and values of u_1 and u_2 are calculated, they are rounded up or down to the nearest value of 0.005. The dotted lines in Figure 2 show the results of simulating valve inaccuracies. An oscillation develops in the y_2 response because of the finite resolution of the valves. The oscillation is also similar to what occurs in practice as a result of valve hysteresis. This oscillation has a peak-to-peak value of 0.09 (4.5% of the range of y_i), and it does not die out with time. The oscillation is similar to the type of oscillation one would get in trying to drive an automobile with a very sensitive steering wheel. The oscillation in y_2 is not dependent on the size of the step change in y_1. Even if a small step in the setpoint of y_1 is introduced, oscillations of approximately the same magnitude in y_2 result. Figure 2 also shows that the y_1 response is not that much different from that which one would calculate using a computer. There is a small sustained oscillation in y_1 between 0.22 and 0.23. This small oscillation is transmitted to y_2 through the large K_{21} gain of 100, and this transmission results in the large y_2 oscillations. If σ_1 were much smaller, then the small oscillation would not present a problem. If the resolution of the valves in this example were poorer than ± 0.005, then the magnitude of the oscillations in both variables would increase. For both measurements, the magnitude doubles if the resolution doubles to ± 0.01.

The oscillations shown in Figure 2 can be decreased if the controller for y_1 is detuned. Figure 3 gives results for the case where K_{C1} is reduced to 0.0513 while K_{C2} remains the same. The dotted curves give the response for the detuned system, while the solid curves are the same as those in Figure 2. As can be seen, the detuned responses do not exhibit severe oscillations. However, the performance of the y_1 control system has become much slower because of the detuning, and the time it takes for y_1 to get to its new setpoint is more than twice that for the original tuning. If it were important that y_1 be tightly controlled, then this detuning presents a problem that cannot be avoided if real-world valves are used and oscillation in y_2 cannot be tolerated. In general,

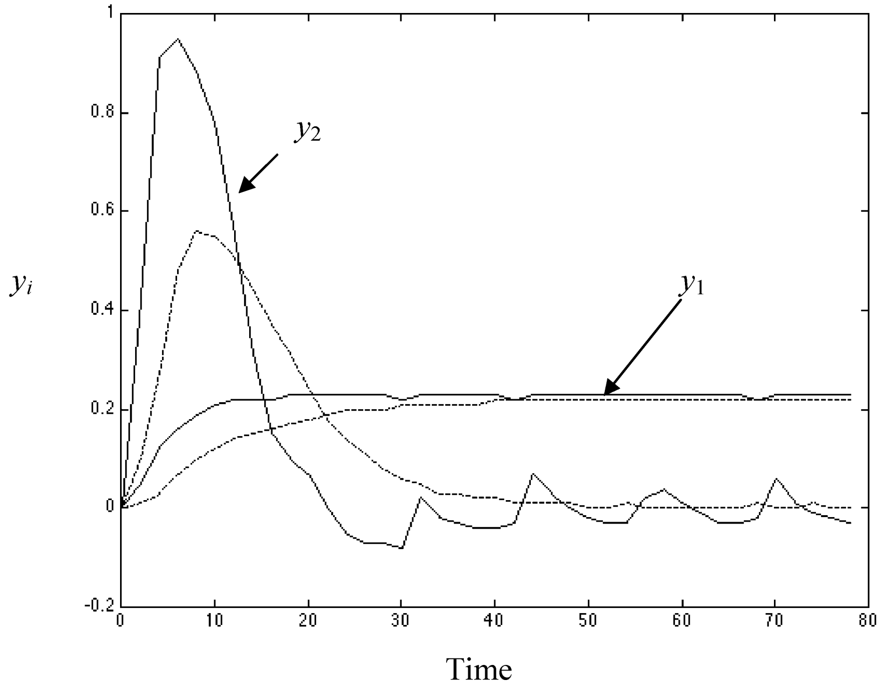

Figure 3. Closed-loop responses with inaccurate valves and initial tuning (solid lines) and inaccurate valves with K_{C1} reduced to 0.0513 (dashed lines).

detuning controllers for systems with large σ_1 should reduce oscillations but with the penalty that the transient performance also will be reduced.

This discussion suggests that it would be reasonable to impose an upper limit on σ_1 for systems with $\sigma_n \geq 1$ when a square controller with integral action is used. Either having sustained oscillations with a significant magnitude in the controlled variables or detuning controllers is not desirable. Because essentially all control systems are eventually implemented with analogue devices, which typically have an accuracy on the order of 0.5% of full scale, a reasonable limit on σ_1 would be 50. One could argue with the exact value of the limit proposed and say for example that it should be 100. However, there should be an upper limit because a system with a σ_1 of 1000 would not work in practice if a square controller with integral action in all channels is used. For such a system, one would not be able to get the fine manipulation of the control valves that would be required for control because the valves would be limited to move in an extremely small region within their range. In the sections below, an approach to selecting measurements and manipulated variables to avoid both actuator and measurement saturation is presented. Before the approach to avoiding both actuator and sensor saturation is pressented, an approach to analyzing measurement correlation is considered first.

Analysis of Correlation among Measured Variables

To analyze measurement correlation in process systems, a new interaction tool is proposed. It can be noted that this tool is general and it can be applied to processes with any singular values and also to nonsquare processes. To use the approach, it is assumed that each of the manipulated variables, u_i, is forced with an independent, zero-mean, unit variance signal. This forcing results in stochastic variations in each of the process outputs, y_i. Depending on $\mathbf{K}$, the y_i responses may be correlated with one another. The coefficient of determination,[14] $r_{i,k}^2$, measures the proportion of the variation in y_i that is explained by y_k. The coefficient of determination is the square of the correlation coefficient, and it ranges between 0 and 1. Clearly, if two outputs are tightly correlated, then it will be extremely difficult to control each output independently. Further, a disturbance in one output could be transferred to the other through the control system. These points are illustrated in Figure 4, where two variables, y_1 and y_2, that are correlated with different $r_{1,2}^2$ values are plotted. To generate this plot, independent random values of y_1 are calculated and then the correlation coefficient was used to calculate y_2. As $r_{1,2}^2 \to 1$, the behavior of y_2 follows the behavior of y_1 more and more closely. In the employment of $r_{1,2}^2$ to decide on the feasibility of control system architectures, it is necessary to choose a threshold. In this paper, a value of 0.8 is used for this threshold, although its exact value is a judgment call.

It is interesting that the $r_{i,k}^2$ values depend only on the $\mathbf{\Sigma}$ and $\mathbf{U}$ matrices of SVD of $\mathbf{K}$. Under the stated assumptions on u,

$$\mathrm{cov}(y) = \mathrm{cov}(\mathbf{K}u) = \mathbf{K}\,\mathrm{cov}(u)\,\mathbf{K}^{\mathrm{T}} = \mathbf{K}\mathbf{K}^{\mathrm{T}} = \mathbf{U}\Sigma\mathbf{V}^{\mathrm{T}}\mathbf{V}\Sigma^{\mathrm{T}}\mathbf{U}^{\mathrm{T}} = \mathbf{U}\Sigma\Sigma^{\mathrm{T}}\mathbf{U}^{\mathrm{T}} = \mathbf{B}\mathbf{B}^{\mathrm{T}} \quad (14)$$

where $\mathbf{B} \equiv \mathbf{U\Sigma}$. Hence, the expression for the coefficient of determination, $r_{i,k}^2$, in ref 14:

$$r_{i,k}^2 = \frac{(\sum_j K_{i,j}K_{k,j})^2}{(\sum_j K_{i,j}^2 \sum_j K_{k,j}^2)}$$

remains correct, with the elements of $\mathbf{K}$ replaced by the

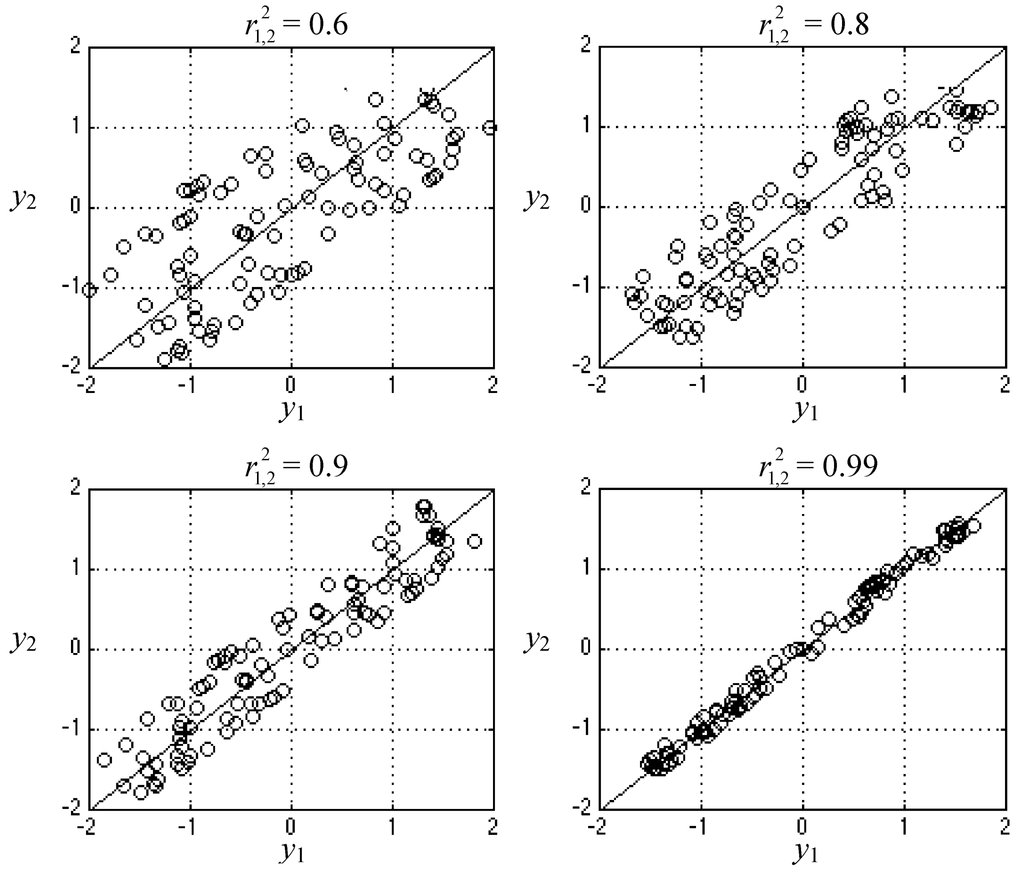

Figure 4. Correlations between two measured variables for various values for the correlation coefficient.

corresponding elements of **B**:

$$r_{i,k}^{\ 2} = \frac{(\sum_j b_{i,j} b_{k,j})^2}{(\sum_j b_{i,j}^{\ 2} \sum_j b_{k,j}^{\ 2})} \tag{15}$$

where $b_{i,j}$ are the elements of **B** and j is summed from 1 to the number of manipulated variables. To evaluate control system feasibility, the $r_{i,k}^2$ elements are arranged into a matrix, **R**. Processes with large CN can give rise to $r_{i,k}^2$ values that are close to 1. To configure square control systems, $r_{i,k}^2$ can be used to eliminate measurements and improve the CN for the process. Before an illustration of how this elimination can be carried out is given, three cases are considered first.

Case 1: $\sigma_1 = 1$. The system in this case has the best possible CN, and the $b_{i,j}$ elements are equal to $u_{i,j}$. Because **U** is a unitary matrix and its columns are orthogonal,

$$\mathbf{R} = \mathbf{I} \tag{16}$$

That is, there is no measurement correlation in this case.

Case 2: Large σ_1 or Small σ_n and U = I. In this case, **B** is a diagonal matrix. Substitution into eq 15 again results in eq 16, and there is no measurement interaction. An example for this case is

$$\mathbf{K} = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} \tag{17}$$

for which σ_1 is 100. The gain of 100 should be questioned if this were a real system. Such a gain means that u_1 has an extremely large effect on y_1. The large gain could arise if too large a valve or too small a transmitter for y_1 is used, and the instrumentation could be changed and no doubt a better control performance would result. If neither of these cases holds, then the system given by eq 17 would be somewhat unusual. It would have valves and transmitters with reasonably installed spans, yet u_1 would have an extremely large effect on y_1.

Case 3: Large σ_1 and U Is Significantly Different from an Identity Matrix. If σ_1 is very large relative to the other σ_i and none of the elements of u_1 are zero, then the first column of **B** will dominate the calculation of **R**. As a result, **R** will approach

$$\mathbf{R} \simeq \begin{bmatrix} 111... \\ 111... \\ 1......... \\ 1......... \end{bmatrix} \tag{18}$$

and all measurements will be nearly perfectly correlated. Equation 11 gives an example of a system that falls into case 3. The **U** and **Σ** matrices for this system are

$$\mathbf{U} = \begin{bmatrix} 0.1081 & -0.9941 \\ 0.9941 & 0.1081 \end{bmatrix} \tag{19}$$

$$\mathbf{\Sigma} = \begin{bmatrix} 101.20 & 0 \\ 0 & 1.196 \end{bmatrix} \tag{20}$$

Substitution into eq 15 gives

$$\mathbf{R} = \begin{bmatrix} 1.000 & 0.988 \\ 0.988 & 1.000 \end{bmatrix} \tag{21}$$

indicating that the two outputs are closely correlated.

Generating Control System Architectures with Good Singular Values

In this section it is assumed that control systems with σ_i in the range

$$1 \le \sigma_i \le 50 \tag{22}$$

are desired. Two simple systems are treated first, and then the approach is applied to the Tennessee Eastman process.[13] If eq 22 is satisfied, then actuator saturation will be avoided. However, it is possible that the resulting control system might not be the best possible design. If an effective override scheme can be found when one or more actuators approach saturation, then it may be that the corresponding scheme could exhibit a superior dynamic performance. An advantage of developing plantwide schemes using eq 22 is that they will not require overrides and therefore they will be simple. The issue of overrides will be addressed again when the Tennessee Eastman process[13] is considered.

To select measurements and manipulated variables, the scaled process gain matrix will be split, as shown in Figure 5. The vector of manipulated variables, u, is first rotated using $\mathbf{V}^T$ to produce $\boldsymbol{\mu}$ as

$$\boldsymbol{\mu} = \mathbf{V}^T u \tag{23}$$

The rotated manipulated variables, μ_i, and $\boldsymbol{\Sigma}$ can be used to eliminate weak manipulated variables. For an $n \times n$ system, μ_n will act through the smallest singular value, and therefore it will have the smallest effect on the measured variables. In general, the u_i with the largest coefficient in the last row of $\mathbf{V}^T$ would be eliminated because it would have the largest contribution to μ_n. However, one also has to look at the upper rows to make sure that no large values occur there. With the approach discussed below, one can use $\mathbf{V}^T$ to suggest manipulated variables to be eliminated and then check SVD for the reduced gain matrix to make sure that the minimum singular value is greater than 1. Thus, the approach discussed here gives guidelines as to how to choose variables so that a reduced dimension process with acceptable singular values results. The number of manipulated variables eliminated would be equal to the number of singular values that are less than 1. Next, the μ_i values are used to calculate y as

$$y = \mathbf{U}\boldsymbol{\Sigma}\boldsymbol{\mu} \tag{24}$$

Correlation analysis is used to eliminate outputs with $r_{i,k}^2 \ge 0.8$ to produce a final system with the σ_i values in the desired range of 1−50. After considering two simple examples, the methodology is applied to the Tennessee Eastman process.

Example 1. Consider a process with the following scaled gain matrix and assume that a square control system is desired:

$$\mathbf{K} = \begin{bmatrix} 0.6141 & 0.8927 & -0.0049 \\ 0.6658 & 0.9421 & 0.0092 \\ 1.1961 & -0.9661 & 0.0124 \end{bmatrix} \tag{25}$$

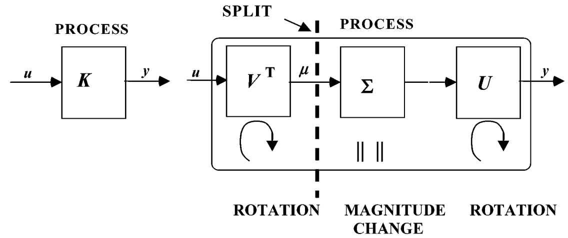

Figure 5. Splitting the process gain matrix using SVD.

SVD can be used to determine how many control loops can be closed for this system, and if variables must be eliminated, the $\mathbf{V}^T$ and $\mathbf{R}$ matrices can be used to determine the final control system architecture. It is obvious from the gain matrix that u_3 has a very small effect on any of the three measured variables. SVD of $\mathbf{K}$ is

$$\mathbf{U} = \begin{bmatrix} 0.5713 & 0.3767 & 0.7292 \\ 0.6035 & 0.4094 & -0.6843 \\ -0.5563 & 0.8310 & 0.0066 \end{bmatrix} \tag{26}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1.6183 & 0 & 0 \\ 0 & 1.5000 & 0 \\ 0 & 0 & 0.0098 \end{bmatrix} \tag{27}$$

$$\mathbf{V}^T = \begin{bmatrix} 0.0539 & 0.9985 & -0.0026 \\ 0.9985 & -0.0539 & 0.0081 \\ 0.0080 & -0.0030 & -1.0000 \end{bmatrix} \tag{28}$$

Examination of the $\boldsymbol{\Sigma}$ matrix shows that σ_3 is much smaller than 1 and actuator saturation is a potential problem if three manipulated variables are used. The fact that one singular value is less than 1 indicates that one manipulated variable needs to be eliminated to guarantee that actuator saturation does not occur. The weakest manipulated variable is μ_3 because it acts through σ_3. The $\mathbf{V}^T$ equation shows that μ_3 is essentially $-u_3$ in this case, and thus u_3 is eliminated. The coefficients for u_3 in the first and second rows of $\mathbf{V}^T$ are small, so u_3 does not contribute much through σ_1 and σ_2. Once u_3 is eliminated, SVD of the resulting 2 × 3 matrix can be calculated. The resulting two singular values are almost identical to σ_1 and σ_2 in eq 27. The fact that both of these singular values are greater than 1 and neither is greater than 50 indicates that two loops can be closed and normal controller tuning can be used. However, because the singular values can change when variables are eliminated, one needs to check SVD for the reduced process to verify that both singular values satisfy eq 22. Next the $r_{i,k}^2$ matrix can be calculated from the 2 × 3 matrix by using eq 15 to give

$$\mathbf{R} = \begin{bmatrix} 1.0000 & 0.9998 & 0.0059 \\ 0.9998 & 1.0000 & 0.0041 \\ 0.0059 & 0.0041 & 1.0000 \end{bmatrix} \tag{29}$$

The $\mathbf{R}$ matrix indicates that outputs y_1 and y_2 are very tightly correlated. In the original gain matrix, row 2 is almost a constant multiple of row 1, and this fact results in the high correlation between outputs y_1 and y_2. Thus, to produce a square process gain matrix that has singular values between 1 and 50, one needs to eliminate either y_1 or y_2 and then check SVD of the resulting gain matrix. If y_1 and y_3 are controlled with u_1 and u_2 or y_2 and y_3 are controlled with u_1 and u_2, then the

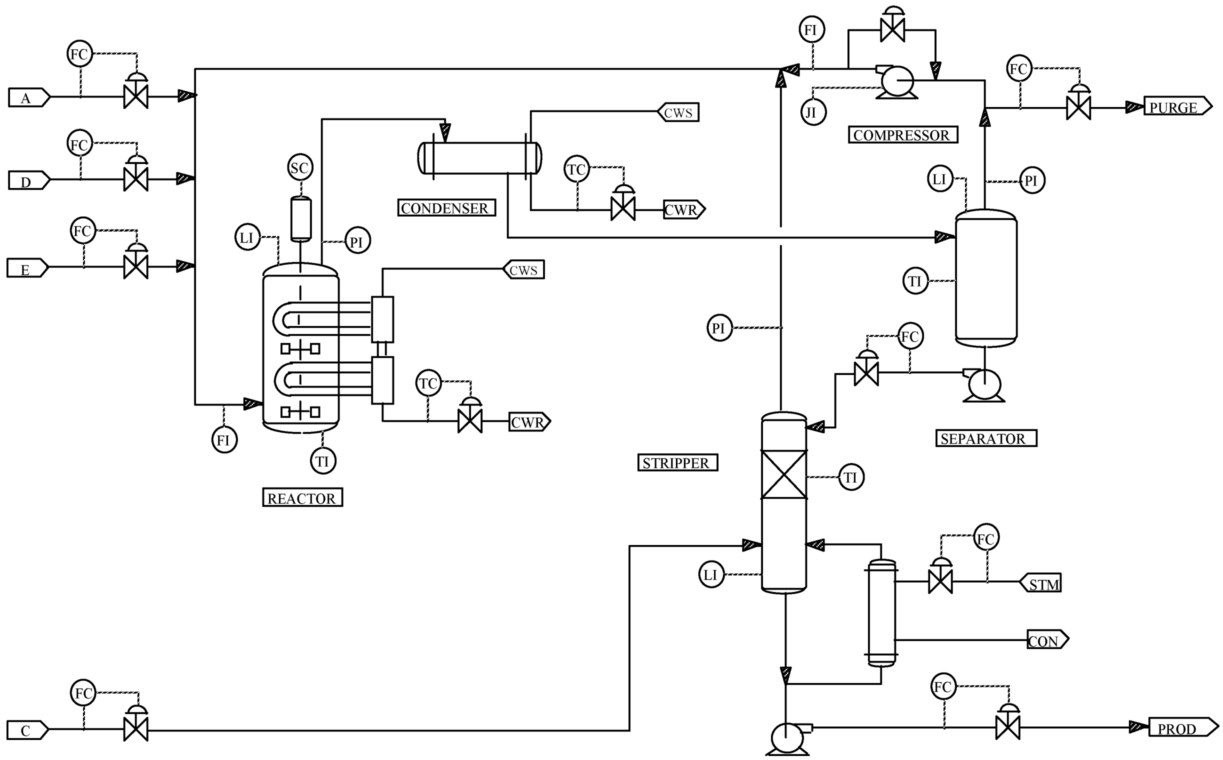

Figure 6. Tennessee Eastman process with inner cascade loops closed.

values for σ_i are acceptable. Control of y_1 and y_2 with u_1 and u_2 gives a CN of 158 because in this case $\sigma_2 = 0.01$.

Example 2. Consider a system with the following scaled gain matrix:

$$\mathbf{K} = \begin{bmatrix} -0.7963 & -0.7982 & -0.8052 \\ -1.0774 & -0.0821 & -0.0719 \\ -0.9078 & 0.7976 & 0.7915 \end{bmatrix} \quad (30)$$

SVD can be used to determine how many control loops can be closed for this system, and if variables must be eliminated, the $\mathbf{V}^T$ and $\mathbf{R}$ matrices can be used to decide on a final control system architecture. If SVD of $\mathbf{K}$ is calculated, the $\mathbf{\Sigma}$ matrix given by eq 27 results, except with $\sigma_2 = 1.6000$. The $\mathbf{U}$ and $\mathbf{V}^T$ matrices are given by

$$\mathbf{U} = \begin{bmatrix} -0.4931 & -0.7079 & 0.5057 \\ -0.6658 & -0.0671 & -0.7431 \\ -0.5599 & 0.7031 & 0.4383 \end{bmatrix} \quad (31)$$

$$\mathbf{V}^T = \begin{bmatrix} 1.0000 & 0.0010 & 0.0011 \\ -0.0015 & 0.7071 & 0.7071 \\ 0.0000 & 0.7071 & -0.7071 \end{bmatrix} \quad (32)$$

To eliminate a manipulated variable in this case, one has two choices. Both u_2 and u_3 have coefficients with a magnitude of 0.707 in row 3 of $\mathbf{V}^T$. One of these manipulated variables can be eliminated and the other retained. The $\mathbf{V}^T$ matrix shows that it is the difference $u_2 - u_3$ that acts through the smallest singular value. If u_3 is eliminated, the $\mathbf{R}$ matrix that results for the 3 $\times$ 2 system is

$$\mathbf{R} = \begin{bmatrix} 1.0000 & 0.5746 & 0.0040 \\ 0.5746 & 1.0000 & 0.4885 \\ 0.0040 & 0.4885 & 1.0000 \end{bmatrix} \quad (33)$$

Equation 33 indicates that any of the three measurements can be used because they are not highly correlated. Eliminating y_2 is preferred because then both singular values are between 1 and 2 (also $\mathbf{R} \approx \mathbf{I}$). Eliminating either of the other two manipulated variables results in a singular value of less than 1. Next, the methodology is applied to the Tennessee Eastman process.

Application to the Tennessee Eastman Process

The SVD approach can be applied to the Tennessee Eastman testbed process,[13] with a schematic diagram of the plant shown in Figure 6. The case considered is the base case.[13] There are 10 inner cascade loops that can be closed in the Eastman plant. These involve 8 flow controllers and 2 cooling water temperature controllers. In the gain calculations below, these controllers are assumed to be operational. A nonlinear state-space model of the form

$$\dot{x} = f(x,u) \quad (34)$$

$$y = g(x,u) \quad (35)$$

where x are the process states, u is the vector of manipulated variables, and y is the vector of measured variables is made available by the authors. The nonlinear functions f and g in eqs 34 and 35 can be linearized around the steady-state operating point using numerical differentiation to give

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \quad (36)$$

$$y = \mathbf{C}x + \mathbf{D}u \quad (37)$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are constant matrices. Because the model contains pure integrating elements associated with the separator and stripper levels, calculation of a steady-state gain matrix is not straightforward. Arkun

Table 1. Measured and Manipulated Variables

process measurements	manipulated variables
recycle flow (RecF)	E feed setpoint (E^{sp})
reactor feed (ReaF)	compressor recycle valve (CRV)
reactor pressure (ReaP)	purge setpoint (Pu^{sp})
reactor level (ReaL)	separator exit flow setpoint (Se^{sp})
reactor temperature (ReaT)	product-flow setpoint (Pr^{sp})
separator temperature (SepT)	steam-flow setpoint (St^{sp})
separator level (SepL)	reactor cooling water temperature setpoint (RCT^{sp})
separator pressure (SepP)	condenser cooling water temperature setpoint (CCT^{sp})
stripper level (StrL)	agitator (Ag)
stripper pressure (StrP)	
stripper temperature (StrT)	
compressor power (CP)	

Table 2. Scale Factors

variable type	scale factor	comments
flows	2(steady-state flow)	
pressures	600 kPa	
temperatures	50 °C	
compressor power	2 × steady state	
reactor level	50%	not integrating
other levels	50%/h	integrating
manipulated variables	2(steady-state value)	steady state < 50%
manipulated variables	2(100% steady-state value)	steady state > 50%

and Downs[15] proposed an approach to overcome this problem. Their method results in a gain matrix that involves the rate of change of the two integrating levels together with the normal gains of the remaining 39 measurements.

To illustrate the use of SVD and correlation analysis to determine a plantwide architecture, a control system for the noncomposition variables is examined. After the 10 inner cascade controllers are closed, there are 12 noncomposition measurements that potentially can be controlled, and these are shown in Table 1. In the statement of the Tennessee Eastman problem,[13] three flows (A, D, and C feeds) have constraints on how fast they can be manipulated. Because the noncomposition variables involve some fast loops, these three setpoints are eliminated from the analysis. As a result, there are nine potential manipulated variables, and these are also shown in Table 1.

After the process gain matrix is calculated, it has to be scaled to be used in an SVD analysis. Scaling of the manipulated variables is carried using information provided in the reference.[13] All of the manipulated variables are specified in units of percent of full scale. If the steady state percent value is less than 50%, a scale factor of twice the steady-state value is used. If the steady state percent value is greater than 50%, then a scale factor of twice the difference between 100 and the steady-state value is used. This scaling allows for a manipulated variable to move in a range such that it will just saturate either open or closed. Scaling the process measurements is more difficult because scale factors for these variables are not provided in the reference.[13] In the calculations, measurements were scaled using the scale factors given in Table 2. Flows and compressor power are scaled by twice their steady-state values. The steady-state reactor pressure is 2705 kPa, and the plant shuts down if this pressure reaches 3000 kPa. The scale factor for the three pressure measurements is set at 600 kPa, which is approximately twice the shutdown range [2(3000 − 2705)]. Temperatures are scaled for a 50 °C range. The reactor level is nonintegrating, and it is scaled for a range of 50%. The other two levels are integrating, and the gain matrix gives their rate of change. These levels are scaled using 50%/h as a scale factor.

Chart 1

$\mathbf{V}^T$ =

E^{sp}	CRV	Pu^{sp}	Se^{sp}	Pr^{sp}	St^{sp}	RCT^{sp}	CCT^{sp}	Ag
0.9347	**0.0660**	**0.0297**	**0.3202**	**0.0720**	**0.0015**	**-0.0296**	**0.1110**	**0.0115**
-0.1795	0.0490	-0.0035	0.6466	-0.7338	-0.0005	-0.0432	0.0815	0.0167
0.2957	-0.0667	-0.0050	-0.6477	-0.6692	0.0145	0.1654	-0.0954	-0.0640
-0.0237	-0.0936	0.2648	0.1821	0.0856	0.0084	0.8663	-0.1310	-0.3353
0.0566	-0.1972	0.4047	0.1024	-0.0137	-0.1540	-0.2501	-0.8294	0.0968
-0.0157	***0.9532***	**-0.0564**	**-0.0388**	**-0.0028**	**-0.0173**	**0.0745**	**-0.2826**	**-0.0288**
0.0371	**-0.1293**	***-0.7024***	**0.0759**	**0.0184**	**-0.6396**	**0.1353**	**-0.2285**	**-0.0524**
-0.0354	**0.1262**	**0.5183**	**-0.0947**	**-0.0240**	***-0.7527***	**-0.0527**	**0.3673**	**0.0204**
0.0000	**-0.0000**	**-0.0000**	**0.0000**	**0.0000**	**0.0000**	**-0.3609**	**-0.0000**	***-0.9326***

After the process gain matrix is scaled, its SVD is calculated as $\mathbf{U\Sigma V}^T$. First, consider $\mathbf{\Sigma}$, which has the following elements:

$$\mathbf{\Sigma} = \mathrm{diag}[75.33\ 26.09\ 8.545\ 4.819\ 1.475\ 0.379\ 0.327\ 0.198\ 4.59 \times 10^{-12}] \quad (38)$$

Only five of the singular values are greater than 1.0, which indicates that roughly five variables could be controlled without potential actuator saturation. The reason that the indication is rough is that the singular values can change when variables are eliminated. Thus, there may be more, or possibly less, singular values that are greater than 1 after manipulated and measured variables are eliminated. Whether such an increase occurs can be checked by applying SVD to the reduced matrix. The large σ_1 indicates that valve accuracy could be a problem, and some controller detuning may be required. Thus, the $\mathbf{\Sigma}$ matrix gives a preliminary indication that 5 loops can be closed but detuning may be necessary because of the singular value of 75.33. The $\mathbf{V}^T$ matrix can be used to eliminate manipulated variables that have most of their effect through the singular values that are less than 1.0. It can also be used to determine which manipulated variable acts through the largest singular value. The $\mathbf{V}^T$ matrix, where the first and last four rows are shown in boldface, is given in Chart 1. The first row involves the linear combination that acts through σ_1. The last four rows involve the linear combinations of manipulated variables that act through the four smallest σ_i. In each of the boldface rows, the element with the largest coefficient is shown in underlined italics. Considering the last row of $\mathbf{V}^T$, it can be seen that the agitator, manipulated variable 9, has a coefficient of 0.9326. Although the agitator does have a nonzero coefficient in the fourth row, it is a weak manipulated variable and can be eliminated. The next to last row shows that manipulated variable 6, the steam-flow setpoint, has the largest coefficient and can be eliminated. It can be noted that when Ricker carried out a steady-state optimization on the Tennessee Eastman plant,[16] he concluded that the steam should be shut off. The sixth and seventh rows indicate that manipulated variables 2 and 3, the compressor recycle valve and the purge flow, contribute the most through σ_6 and

Chart 2

R =

RecF	ReaF	ReaP	ReaL	ReaT	SepT	SepL	SepP	StrL	StrP	StrT	CP
1.0000	0.9970	**0.9281**	**0.8659**	0.6226	0.9683	0.4173	0.9281	0.2564	0.9278	0.9528	0.6444
0.9970	1.0000	**0.9526**	**0.8994**	0.5862	0.9794	0.4370	0.9527	0.2671	0.9524	0.9713	0.5918
0.9281	0.9526	**1.0000**	**0.9890**	0.3874	0.9466	0.4849	1.0000	0.2905	1.0000	0.9723	0.3790
0.8659	0.8994	**0.9890**	**1.0000**	0.2971	0.9014	0.5160	0.9889	0.3055	0.9891	0.9417	0.2835
0.6226	0.5862	0.3874	0.2971	**1.0000**	0.6075	0.1228	0.3875	0.0848	0.3871	0.5295	0.8135
0.9683	0.9794	**0.9466**	**0.9014**	0.6075	1.0000	0.4444	0.9466	0.2714	0.9465	0.9937	0.5186
0.4173	0.4370	0.4849	0.5160	0.1228	0.4444	**1.0000**	0.4849	0.5597	0.4851	0.4695	0.1272
0.9281	0.9527	**1.0000**	**0.9889**	0.3875	0.9466	0.4849	1.0000	0.2905	1.0000	0.9723	0.3792
0.2564	0.2671	0.2905	0.3055	0.0848	0.2714	0.5597	0.2905	**1.0000**	0.2906	0.2843	0.0864
0.9278	0.9524	**1.0000**	**0.9891**	0.3871	0.9465	0.4851	1.0000	0.2906	1.0000	0.9723	0.3784
0.9528	0.9713	**0.9723**	**0.9417**	0.5295	0.9937	0.4695	0.9723	0.2843	0.9723	1.0000	0.4490
0.6444	0.5918	0.3790	0.2835	**0.8135**	0.5186	0.1272	0.3792	0.0864	0.3784	0.4490	1.0000

σ_7 and can be eliminated. It should be noted that Ricker[17] has presented a very effective plantwide control scheme for the Tennessee Eastman process that used the purge for reactor pressure control. However, his scheme required an override approach when the purge stream approached saturation. Eliminating the purge ensures that actuator saturation is not a problem at steady state, but it may be that using an override approach might give rise to a better dynamic plantwide control system.

Now consider the first row, which gives the linear combination of manipulated variables that acts through the largest singular value. By far the largest contributor to this row is manipulated variable 1, the E feed setpoint. If a single-input single-output plantwide control scheme is used, then the loop that involves the E feed may need to be detuned because of valve accuracy considerations. The manipulated variables that remain after elimination of weak manipulated variables are the E feed setpoint, separator exit, and product-flow setpoints and the condenser and reactor cooling water temperature setpoints. The singular values calculated for this 12 × 5 matrix are

$$\mathbf{\Sigma} = \text{diag}[75.13\ 26.06\ 8.51\ 4.33\ 1.29] \qquad (40)$$

Note that these singular values are very close to the first five singular values in eq 38. To check whether addition of one of the manipulated variables that was eliminated can result in a sixth singular value greater than 1, each manipulated variable can be added one at a time, but this addition does not yield a sixth singular value greater than 1. If the purge is added, then the first five singular values are close to those in eq 40 and the sixth singular value is 0.283. It should be emphasized again that eq 9 involves the worst-case setpoint change or disturbance and the actual disturbances encountered may not cause actuator saturation.

Next the **R** matrix is calculated using eq 15 on the 12 × 5 process gain and is given in Chart 2. Once the number of loops is determined to be 5, then the possibilities for controlling the available 12 measured variables are limited. In most cases, it is desirable to control the liquid levels in a process to avoid a spill or to have the level run dry. In the Tennessee Eastman process, controlling the reactor pressure and temperature is also desirable because the reactor is open-loop unstable. At this point, there are five potential measured variables to control, and because five manipulated variables are available, the remaining measured variables are not considered. Elements greater than 0.8 in the columns for the three levels and two reactor measurements are shown in boldface in Chart 2. The following observations can be made about the five measurements. First, the separator level, stripper level, and reactor temperature are only weakly correlated with the remaining two measured variables. The reactor level and pressure are tightly correlated with one another, and they are correlated with the recycle flow, reactor feed, and separator and stripper temperatures and pressures. Thus, one can expect some difficulty in attempting to control these two measured variables simultaneously, and loops involving them may need to be detuned.

RGA for the 5 × 5 system that results from applying the methodology discussed in this paper is

$$\begin{array}{c} \\ \Lambda = \end{array} \begin{array}{c} \begin{array}{ccccc} E^{SP} & Se^{SP} & Pr^{SP} & RCT^{SP} & CCT^{SP} \end{array} \\ \left[\begin{array}{ccccc} -1.3846 & -1.3002 & -0.0000 & 0.0521 & 3.6328 \\ 1.7842 & 1.7178 & 0.0000 & 0.0213 & -2.5234 \\ 0.1059 & 0.0813 & -0.0000 & 0.9291 & -0.1163 \\ 0.4945 & 0.5011 & -0.0000 & -0.0025 & 0.0069 \\ 0.0000 & -0.0000 & 1.0000 & 0.0000 & -0.0000 \end{array}\right] \end{array} \begin{array}{l} \\ \text{ReaP} \\ \text{ReaL} \\ \text{ReaT} \\ \text{SepL} \\ \text{StrL} \end{array}$$

The first two rows indicate that the reactor pressure and level loops will exhibit interaction. Rows 3 and 5 indicate that the reactor temperature and stripper level are essentially decoupled from the other loops. The RGA analysis of these four rows agrees with the results predicted by the correlation matrix. Row 4 of RGA indicates that the separator level will also exhibit interaction, but the correlation matrix indicates that the separator level is weakly correlated with the other four measurements. At present, there are no insightful mathematical relationships between the correlation matrix and RGA. The correlation approach is scale-dependent and RGA is scale-independent, so there may not be a simple insightful relationship. Indeed, SVD can give different implications and insights than the RGA, and there is no one-to-one mapping between the SVD and RGA. If a process has large off-diagonal RGA elements, then it will be poorly conditioned, but the converse is not true.[9] A process can be poorly conditioned even though it has small off-diagonal RGA elements.

Summary and Conclusions

This paper has presented an analysis of systems with large singular values, including processes with large CNs where the minimum singular value is greater than 1 so that manipulated variable saturation is not an issue. It has been shown that such systems can be prone to sensor saturation, unless the range of manipulated variable movement is severely limited. It is further shown that limiting manipulated variable movement can lead to poor transient performance because of valve accuracy considerations. This result suggested that practical control systems should have a limit on their maximum singular value. This paper has proposed a value of 50 for the maximum, but the exact value of the maximum is a judgment call. Correlation analysis has been introduced as a tool to help select measured and manipulated variables that have acceptable singular values. The proposed approach has been illustrated on two simple examples and on the Tennessee Eastman process.

It is important to consider several alternatives when using the approach discussed in this paper. Because the gain analysis involves only steady-state information calculated from a linearized model, it may not result in the best overall control architecture when dynamics and nonlinearities are taken into account. It is straightforward to extend the approach presented to the frequency domain in order to take dynamics into account, using the general norms in ref 19. Further, it is straightforward to replace the notion of singular values with the more general notions in refs 11 and 19, e.g., to explicitly consider other norms on the manipulated and measured variables. The methodology discussed in this paper does allow one to narrow in on potential architectures based on the process gain matrix that do not require overrides. However, it may be that a control system that uses overrides can perform better than one that does not. Dynamic simulation can be used to compare alternative control systems. Last, one could easily incorporate engineering judgment in the proposed methodology and select measurements and/or manipulated variables based on experience and process insights.

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