Distribution and Detectability of Dark Matter in the Present Universe

Marissa Cevallos

under the direction of
Dr. Edmund Bertschinger
Massachusetts Institute of Technology

Research Science Institute
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Abstract

The existence of dark matter is one of the largest unanswered questions in cosmology. Dark matter has yet to be detected in a lab setting, even though it is theorized that ninety percent of matter in the universe is dark matter. Understanding how dark matter is distributed in the universe is crucial to detecting it; detecting dark matter is crucial to understanding the underlying physical laws of the universe.

1 Introduction

1.1 What is dark matter?

Dark matter constitutes over ninety percent of the matter in the universe. Sheets of dark matter pass unnoticed through galaxies, rarely colliding with non-dark matter. In fact, dark matter does not respond to the strong or the electromagnetic forces; thus, the electromagnetically interacting photons that we use to probe every type of matter will not interact with dark matter. Indeed, to date, there have been no experimental detections of dark matter. Why should we steadfastly believe in the existence of dark matter when the experimental evidence for ghosts is just as good?

The necessity for dark matter in the universe was realized in the seemingly unrelated observation of orbital velocities in rotating galaxies. The classical approach to celestial rotation assumed that mass was concentrated at the center of the orbit and satellites obeyed Kepler's Laws. By the early 1980's, better cosmological technology enabled scientists to collect experimental evidence about the rotation of spiral galaxies. However, the orbital velocities measured did not match classical predictions: there is not enough visible mass in the galaxy to account for the velocities.

Dark matter has never been directly detected. However, cosmological theory has provided a few guesses as to the nature of the dark matter particle. Dark matter is thought to exist as one of two elementary particles: the neutralino (popularly referred to as a WIMP, a Weakly Interacting Massive Particle), and the axion. The neutralino is theorized to be 1-1000 times more massive (1-1000 GeV) than a proton [2]. The axion is much smaller: its mass is equivalent to 10^{-5} eV, or 10^{-14} times that of the proton. However, neither the neutralino nor the axion has been detected. Dark matter interacts with dark matter and other matter mostly via the gravitational force. However, both neutralinos and axions are predicted to have very weak interactions with other particles.

To detect a dark particle, just as detecting any particle, one would wait until such a particle entered a detection device and interacted with a non-dark particle. One could then measure the effects of the scattering on the hit particle, and infer the properties of the particle that bumped it. Because the chance of a dark particle colliding with another particle is so low, the detectability of dark matter is dependent on how much dark matter happens to wander into the detector; dark matter will only happen to collide into an Earth detector if dark matter has been distributed amply in our part of the universe.

1.2 Detecting dark matter

One may visualize interacting elementary particles as billiard balls colliding and scattering in a game of pool. Think of the surface of the ball as an outside radius surrounding an infinitesimally small point in the center. For an interaction to occur between two particles, the two need only coincide within each others' cross-sectional radius, not collide head-on. The cross-sectional radius is like a force field around a point where gravitational, electromagnetic, weak, and strong forces can form separate cross-sections to influence other particles. It is important to note that when considering dark matter interactions, only the neutralino can be detected by scattering other atomic nuclei; the axion is much too small to deflect nuclei and thus will not be considered in this paper.

The probability of two particles colliding is the probability that their cross-sections overlap in space, given by a probability density function. The function

$$dP = \sigma n_{DET} dl$$

gives the probability of colliding a particle in a confined space, like a particle detector. From the perspective of one dark matter particle traveling through space, σ represents its own cross-sectional area, n_{DET} represents the number density (number of particles per unit volume) of the material of the detector, and dl corresponds to the length of the detector. If a researcher wished to increase his odds of finding some dark matter, he could perhaps increase the length of his detector. However, one can only make a detector so long; eventually, time becomes the ultimate factor in detecting dark matter.

However, one can look at the probability function from the perspective of the detector. The length, dl, of the detector can be replaced by vdt ($v = dl/dt$), where v is the velocity of the dark particles and t is the time the detector waits for dark particles. The function now looks like

$$dP/dt = \sigma S, \tag{1}$$

where S is the flux density $n_{DM}v$. The quantity $n_{DM}v$ is the flux density of dark matter particles incident on one detector atom. A researcher would obviously not build his detector of a single atom, so he could increase the probability of dark matter detection by multiplying by how many atoms he will actully use in his detector.

Dark matter is predicted to be everywhere in the universe. At the time of the Big Bang, dark matter was uniformly distributed, but has since clumped in certain patterns. The aim of this research is to model these distributions and analyze the flux and probability density function of dark matter to predict whether detecting dark matter in current research settings is feasible.

2 Modeling Dark Matter

Dark matter can be treated as a gas in order to describe its macroscopic properties. With a typical gas, properties such as volume, temperature, and pressure describe a large number of particles and not the motion of individual particles. Because of the peculiar nature of the particles composing dark matter, the macroscopic properties of dark matter are not similar to those of gases in classical physics. With classical gases, particles bump into each other through electromagnetic forces, imparting kinetic energy and momentum. Dark matter particles move under gravity only.

2.1 Dark matter trajectories

To completely describe the motion of a system of particles, the trajectory $\vec{x}(t)$ of each particle is necessary. If the initial position of a particle $\vec{x}(0) = \vec{q}$, then the instantaneous position of a particle is given by $\vec{x}(\vec{q}, t)$ where $\vec{q}$ labels the particle. Because we aim for a simple qualitative model of dark matter, we only consider the particle's trajectory in the x direction, so that the trajectories are described completely by the function $x(q, t)$.

We ask that the model of a particle's trajectory satify two conditions: that it oscillate in simple harmonic motion, and that it obey Hubble's Law for early times or large distances.

Simple harmonic motion is the periodic motion shown by oscillating objects (e.g., springs or pendulums) or rotating objects (e.g., anything traveling in orbit). Because dark matter is self-gravitating matter, it acts as a simple harmonic oscillator. In one dimension, we can imagine dark matter particles in the first moment of the universe to be uniformly distributed in a line. After the big bang, dark matter particles fell through the universe via gravitational attraction. Trace the path of the particle in Fig 1. Particles far from x = 0, taken to be a gravitational center, will accelerate slower than those closer to the center. Once a particle falls into the center, it does not clump with other dark matter as early matter forming stars and planets clumped in the early universe. Dark matter will accelerate past the center, depicted by the negative velocity when the sinosoidal curve dips below the x axis. The matter slows after a distance and turns back around, still under gravitational influence. The dark matter oscillates through the center in simple harmonic motion. Extending the graph of the trajectory to infinite time, one can infer that dark matter will continue to oscillate in simple harmonic motion.

Hubble's Law describes galaxies in an expanding universe. It states that the velocity of a galaxy is proportional to the instantaneous distance of it from a fixed point: $v = Hx$, where H is the Hubble's constant. Thus, the ratio v/x must be constant and not dependent on x. Dark matter does not always act according to Hubble's Law. Hubble's Law describes expanding galaxies; dark matter oscillates. However, for small time t or galaxies far enough away, Hubble's Law may be applied to dark matter.

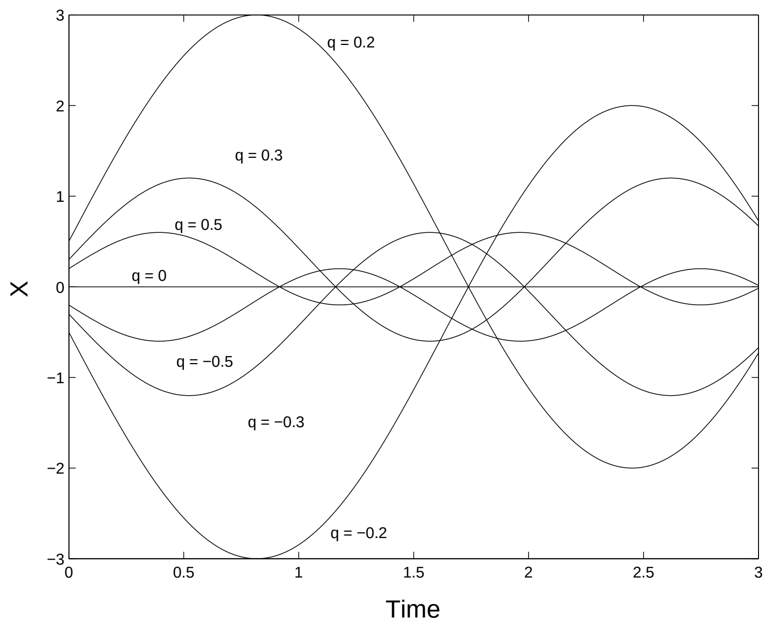


Figure 1: Trajectories for $q = 0, \pm 2, \pm 3, \pm 5$

The graph of

$$x(q,t) = q + Cq^2 \sin qt/(q^2 + a^2) \qquad (2)$$

where a and C are constants, is thus a good model for the behavior of dark matter particles because it obeys intial conditions set by Hubble's law and it predicts that the dark matter will oscillate with simple harmonic motion.

The function corrects for possible division by zero, as well as preventing later graphs derived from trajectories from intersecting themselves (which will be discussed in the section on phase space.)

2.2 Cold collisionless fluid

In the beginning of this Section 2, we discussed why gases are described on the macro scale. However, in 2.1 we modeled individual particles. Why are we describing a large number of particles using their trajectories when classical gases are rarely considered on the molecular scale?

The answer lies in the fact that dark matter is like a cold, collisionless fluid, and not a classical gas. Dark matter composed of neutralinos is "cold" because its particles are massive and move at sub-relativistic speeds. Likewise, hot dark matter refers to very small particles (axions) that move at relativistic speeds. Classical gases are not "cold" gases–rather, they move at high speeds and collide often to transfer heat. In fact, dark matter is cold because it is collisionless. The large-scale properties of a classical gas, like temperature, can be described consistently because the random collisions of all of the particles in a classical gas obey probability.

2.3 Phase space

Phase space refers to a coordinate system in which velocity is plotted against position. To understand how the velocities of dark matter particles are distributed, and how this distribution is different from classical gases, it is useful to translate trajectory to phase space.

Because we will graph position versus velocity, we need an expression $dx/dt = v$. By taking the derivative of $x(t)$ with respect to time we find $v = (Cq^3/(q^2 + a^2)) \cos qt/q^2 + a^2$.

While the graph of the trajectory for this model is similar to that of the simpler model, the graph of its phase space more closely resembles how dark matter is thought to behave [?]. It is important to remember that phase space does not show the path of particles, but rather a distribution of velocities. The diagram of phase space changes with time, as each

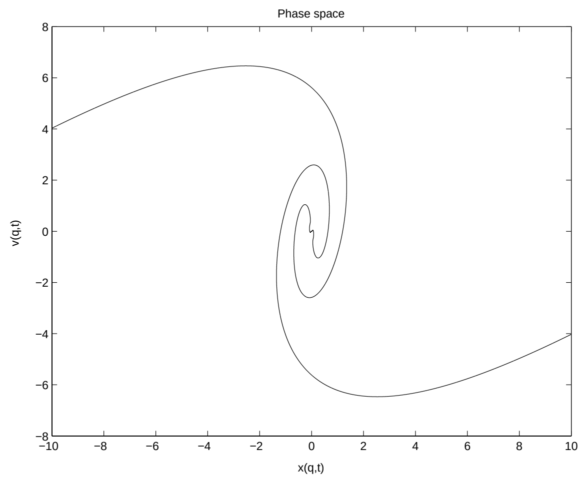
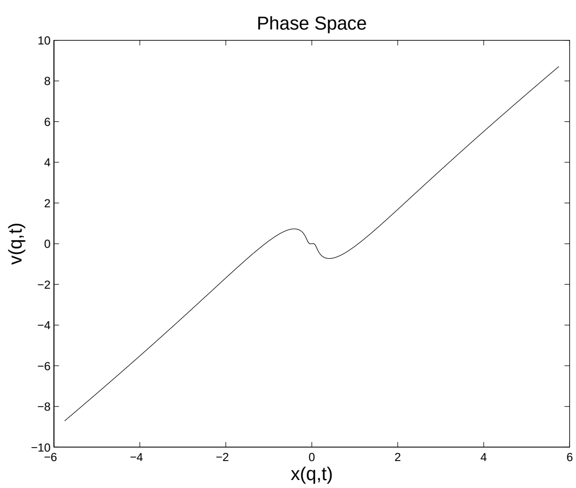


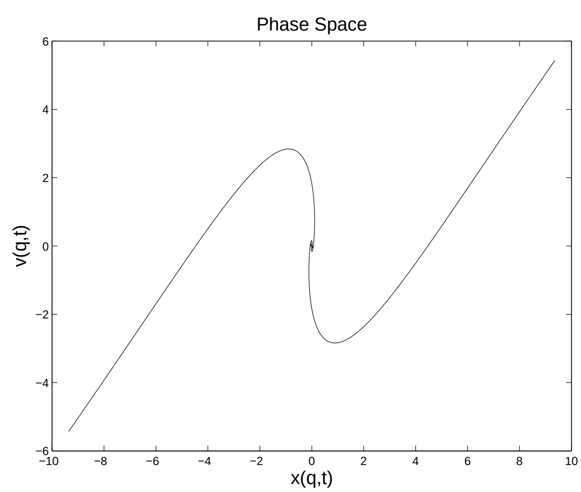
Figure 2: Phase space at time $t = 2$.

individual particle travels with time.

Time $t = 0.5$



Time $t = 1.0$



At t = 0, one can infer graphically that the shape of the phase space plot is the diagonal line $v = Cx$. As the graph of the trajectory oscillates with time, the phase space graph continues to loop from its initial straight line.

After the big bang, at which point dark matter was uniformly distributed, dark matter particles fell through the universe via gravitational attraction.

However, dark matter cannot interact strongly enough to form anything more dense than massive clumps in the universe. Both the neutralino and the axion lack the electromagnetic and nuclear attractions imperative for star and planet formation.

2.4 Number density

This project is concerned with the distribution and flux of dark matter. It is useful to know the number density–the number of particles per unit space–to find the flux of the dark matter, because the flux density is simply the number density times velocity. To model the number density, we contruct a histogram of the number of particles in a number of small "bins" (changes in x.) A program ran through the list of data points ("particles") and sorts them into bins according to their x-coordinate. Though each bin spans the same distance Δx, only at time $t = 0$ will each bin have the same amount of particles. Each particle then moves at a unique velocity and thus the distance between two particles varies.

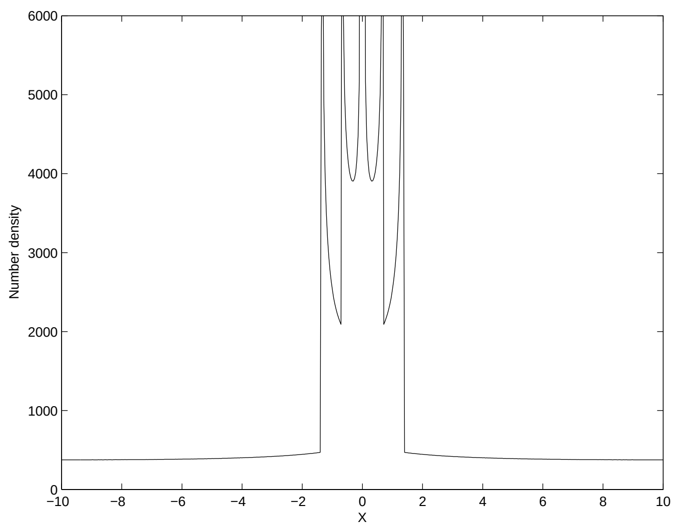


Figure 3: Number density

Figure 4: The number density plot of 500,000 particles in 500 bins at time $t = 2$

The spikes in the graph are referred to as caustics and represent regions of infinite density. Physically, caustics are streams of high density that are separated from low density streams of dark matter. [Zeldovich] The caustics in the number density plot can be predicted from the plot of phase space. The x values at which vertical tangents occur on the plot of phase space correspond to the infinite spikes with number density.

2.5 Flux density

The flux density of dark matter–the product of velocity and the of particles–is the next property we will discuss. The flux density graph is similar to the number density graph: caustics spike at the same x values.

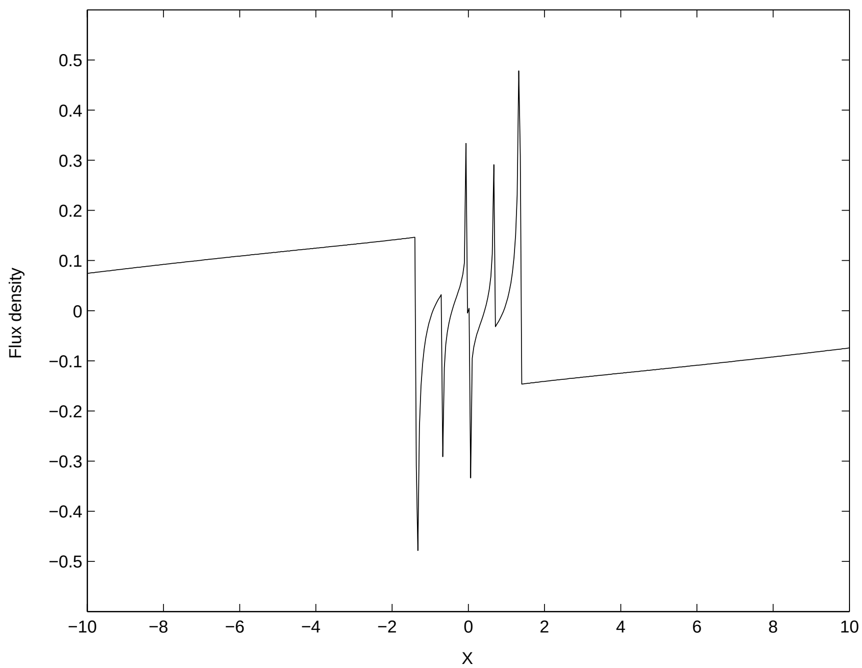


Figure 5: Graph of the flux density at t = 2

3 Dark matter and Maxwell-Boltzmann gases

It is important to emphasize the distinction between a Maxwell-Boltzmann gas and a cold collisionless fluid. The velocities of particles in a Maxwell gas are distributed in a bell curve about $x = 0$; a cold, collisionless fluid has infinitely steep rises and vertical drop-offs. A graph of the trajectories of Maxwell gas particles would consist of a cloud of random points with no obvious connection.

3.1 The Maxwell Boltzmann Distribution

The Maxwell-Boltzmann distribution is a plot in phase space of a classical gas. The derivation for the equation of the curve, which will not be described here, involves partitioning data as we did dark matter. The Maxwell-Boltzmann distribution in one dimension is

$$f(v)dv = \frac{e^{-mv^2/2kT}}{(2\pi kT/m)^{1/2}}dv, \tag{3}$$

where m is the molecular mass of the substance, v is its velocity, k is Boltzmann's constant, and T is the temperature.

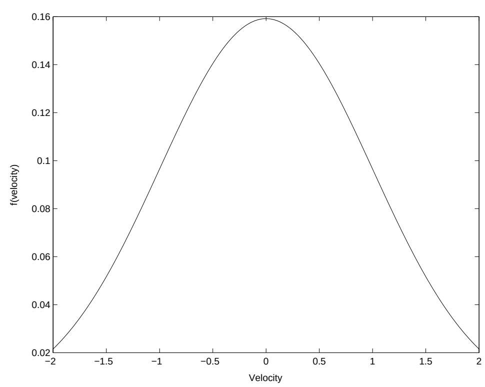


Figure 6: Maxwell-Boltzmann Distribution for $m = kT$

The mean flux density S is given by

$$S = n(x)\langle v \rangle,$$

where $n(x)$ is the number of particles and $< v >$ is the mean velocity. For a Maxwell-

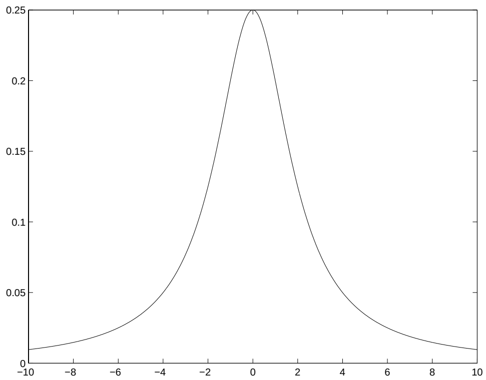


Figure 7: Flux distribution for a Maxwell-Boltzmann gas

Boltzmann gas,

$$< v >= (\frac{2kT}{\pi m})^{1/2}$$

.

Compare the flux distribution of a Maxwell gas with that of a cold, collisionless fluid. The differences are staggering–a Maxwell gas curves smoothly and gradually, while a cold, collisionless fluid has periodic caustics.

4 Conclusion

4.1 Implications on the Detection of Dark Matter

Finding a dark matter particle–a holy grail in current cosmology–is completely dependent on how dark matter is distributed in the universe. The assumption that dark matter behaves as a Maxwell gas leads to grossly different conclusions about its flux distribution than if we assume it is a cold, collisionless fluid. A researcher would not go about looking for a uniformly distributed gas in the same way as one looking for periodic spikes of particles separated by lulls of no activity. This research confirms the results of the 1983 Fillmore and Goldreich paper on the collapse of dark matter. The graphs obtained in this research qualitatively match those that they found.

5 Acknowledgments

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A Title of Appendix

Appendices may appear after the paper proper. Appendices may hold extra information that would interrupt the flow of the paper and that is not absolutely necessary for the reader to appreciate the work. For example, a large number of related figures or a mathematical derivation could go nicely in an appendix.