Comparison of Simulated and Observational Catalogs of Hypervelocity Stars in Various Milky Way Potentials

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Abstract

Hypervelocity stars are stars travelling at velocities great enough to allow their escape from the Milky Way Galaxy. We consider close encounters of binary stars with Sagittarius A*, the massive black hole that is believed to lie at the Galactic center, as the ejection mechanism of hypervelocity stars. We predict the velocity at which these stars are ejected from the Galactic center and their distance from the Galactic center when this happens. After a hypervelocity star is ejected, its orbit is determined by the Milky Way potential. We simulate the behavior of hypervelocity stars in three distinct Milky Way potentials, integrating the orbits over a time shorter than the stellar lifetime. This behavior is then compared to a current survey of 16 hypervelocity stars. We compare the distributions of the rest frame velocity of the hypervelocity stars as well as their distance from the Galactic center. Such a comparison can place constraints on the model of the Milky Way. Similar analyses have been performed by Bromley et al. [1] and Sesana et al [13], but with a smaller catalog of hypervelocity stars. A wider catalog of hypervelocity stars can yield results that are much more refined. In this study, we show how these results can be obtained. Future studies with improved codes and a larger catalog of synthetic stars can yield more accurate statistical analyses and place constraints on the nature of the Milky Way and binary systems.

1 Introduction

In 2005, Brown et al. [2] reported on a star leaving the Milky Way galaxy at a velocity of $v \approx 853$ km s^{-1}, well above its escape velocity from the Milky Way. This observation of a so-called hypervelocity star (HVS) was crucial for providing a strong piece of evidence for the existence of a supermassive black hole (SMBH) at the Galactic Center (GC), specifically a $3.5 \times 10^6 M_\odot$ [1] black hole identified with the radio source Sagittarius A*.

A binary system involves two partners, a primary and secondary, both either a star or black hole. Nearly half of the stars in the Milky Way are a part of a binary system [4]. Yu & Tremaine [14] consider three models for the origin of HVSs involving the interaction between a stellar system and at least one black hole. The first of these models involves an encounter between two stars, the second an interaction between a binary system of two stars and a supermassive black hole, and the third an encounter between a single star and a binary black hole. In this study, we focus on the second model, the tidal breakup of a binary star. In this model, a close encounter between a binary star and a black hole, e.g., Sagittarius A*, leads to an exchange collision.

Hills [10] first suggested that if a binary system passes close enough to a supermassive black hole, that is, within the tidal radius of the binary system, it will experience a perturbation in its orbit that can break it apart. At the tidal radius, the gravitational force between the stars in the binary is equal to the gravitational force on the binary by the SMBH. So, when the binary passes closer to the SMBH than this tidal radius, the gravitational force of the SMBH is great enough to perturb the orbit of the binary and disrupt the binary, causing the two stars to separate in what is called an exchange collision. Hills believed that, if discovered, hypervelocity stars would be sufficient proof of the existence of Sagittarius A*, a SMBH at the GC.

[1] 1 $M_\odot \equiv 1$ solar mass $= 1.988 \times 10^{30}$ kg

At the time that Hills published his work, no hypervelocity stars had yet been discovered. The first hypervelocity star was discovered in 2005 by Brown et al. [2]. This star, SDSS J090745.0+024507, was leaving the Milky Way with a velocity of 853 ± 12 km s^{-1}, the greatest velocity that had yet been measured in the Milky Way. Four years later, a comprehensive catalog of known hypervelocity stars was published by Brown et al.[3]. This catalogue contains 16 known hypervelocity stars and 4 possible hypervelocity stars.

The velocity at which the hypervelocity star is ejected from the MBH is dependent upon a number of parameters, which include the masses of the two stars in the binary (m_1 and m_2), the mass of the black hole ($m_{\rm BH}$), the distance between the binary and the black hole at the time of disruption ($R_{\rm min}$), and the semi-major axis of the binary (a_0). Hills [10], Yu and Tremaine [14], Bromley et al. [1], Sesana et al. [13], and Pfahl [11] found estimates of the theoretical average ejection velocity $\overline{v}_{\rm ejec}$. Bromley et al. [1] and Sesana et al. [13] used empirical distributions of the binary parameters, such as the Salpeter initial mass function [12] and the Heacox [9] distribution for a_0, to numerically generate a catalog of HVSs, both bound and unbound by the Milky Way potential. They also performed calculations to investigate how the Milky Way potential affects the path that the hypervelocity star will travel after ejection.

In the present study, we use similar numerical simulations to create a catalog of synthetic HVSs which we then compare to the current catalog of HVSs. In Section 2, we define the parameters of the synthetic HVSs and create a distribution of the ejection velocities ($v_{\rm ejec}$). We then use these velocities to integrate the orbits of these synthetic HVSs in the Milky Way potential. We do this in three potentials given by Bromley et al. [1] and Dehnen and Binney [5]. In Section 3, we compare our catalog of synthetic stars with the current catalog, attempting to identify the potential that yields orbits most consistent with observations to provide a clearer understanding of models of the Milky Way.

2 Simulations

2.1 Numerical Generation of a Distribution of Ejection Velocities

Our simulation, written in Mathematica version 7.0, considers an initial population of binary stars with the same properties as the initial population used in the study by Bromley et al. [1]. The binaries are initially found at a distance of several thousand AU [2] from the Galactic Center (GC). They are travelling at a velocity of 250 km s^{-1} toward the GC, as suggested by Hills [10]. We assume zero eccentricity.

We first define the characteristic parameters and produce a population of stars using Monte Carlo Methods to produce relevant distributions. For the masses, we adopt the Salpeter [12] initial mass function,

$$p_s(m_1) \sim m_1^{-2.35}, \tag{1}$$

and we assign a value to m_1 in the range 3-10 $M_\odot$, as suggested by current surveys of such stars. We again use the Salpeter IMF to assign a value to m_2 such that .01 $M_\odot \leq m_2 \leq m_1$. We define the more massive star as the primary star and the less massive star as the secondary star.

For the semi-major axes, we use the distribution suggested by Heacox [9],

$$p(a_0) \sim a_0^{-1}. \tag{2}$$

Finally, we find $R_{\min}$ based upon the linear probability density function mentioned by Hills [10]. Observations lead us to assign values to these two parameters such that 0.05 AU $\leq a_0 \leq$ 4 AU and $R_{\min} \leq$ 700 AU.

[2] 1 AU $\equiv$ 1 astronomical unit = 1.49598 $\times 10^8$ km

Having defined all parameters, we are now able to calculate $\overline{v}_{\rm ejec}$. As it can be seen in Appendix A, an analytical estimate of $\overline{v}_{\rm ejec}$ can be found:

$$v_{\rm ejec} \approx (1.49 \times 10^3 {\rm km\ s^{-1}}) \left(\frac{m_{\rm BH}}{10^6 M_\odot}\right)^{\frac{1}{4}} \left(\frac{\rm mpc}{R_{\rm min}}\right)^{\frac{1}{4}} \left(\frac{\delta v}{383 {\rm km\ s^{-1}}}\right)^{\frac{1}{2}}. \tag{3}$$

However, in what follows, we use the distribution suggested by Hills [10], in which

$$\overline{v}_{\rm ejec} = 1760 \left(\frac{a_0}{0.1{\rm AU}}\right)^{-1/2} \left(\frac{m_1 + m_2}{2M_\odot}\right)^{1/3} f_R \ {\rm km\ s^{-1}}. \tag{4}$$

The factor f_R is used to modify $\overline{v}_{\rm ejec}$ so that values are consistent with three-body simulations. Hills [10] defines f_R as

$$f_R = 8.62 \times 10^{-11} D^5 - 4.24 \times 10^{-8} D^4 + 8 \times 10^{-6} D^3 - 6.23 \times 10^{-4} D^2 + 0.0204 D + 0.744 \tag{5}$$

and D as a dimensionless quantity such that

$$D = \left(\frac{R_{\rm min}}{a_0}\right) \left[\frac{2M_{\rm BH}}{10^6(m_1 + m_2)}\right]^{1/3}. \tag{6}$$

For $v_{\rm ejec}$, we assume a Gaussian distribution with dispersion $\delta = 0.2 \cdot \overline{v}_{\rm ejec}$, as published by Bromley et al. [1].

The probability of ejection ($P_{\rm ej}$) of a star in a close encounter with a SMBH, as suggested by Bromley [1], the value of $\frac{R_{\rm min}}{a_0}$. It is defined as

mass ($M_\odot$)	lifetime (Myr)
0.5	20,000
1	9,961
1.25	4,912
1.5	2,695
1.7	1,827
2	1,116
2.5	585
3	352
4	165
5	94
7	43

Table 1: Table of Stellar Lifetimes

$$P_{\text{ej}} \approx 1 - \frac{D}{175}, \tag{7}$$

As negative probabilities are illogical, we reject any stars with a $P_{\text{ej}} \leq 0$. To determine whether or not the star will be ejected, we use another Monte Carlo simulation. We generate a random number between 0 and 1 following a uniform distribution: if this random number is greater than the star's probability of ejection, the star is not ejected. Otherwise, the star is ejected.

We calculate the stellar lifetime of each star through interpolation of empirical data, as recorded in Table 1. We then select a random time at which we assume that the binary encounters Sagittarius A*. If this time is greater than the lifetime of a star with that mass, we deduce that the star does not reach the GC within its lifetime. Therefore, we reject all stars with a stellar lifetime shorter than their approach time.

Finally, assuming that both the primary and secondary stars have equal probabilities of being the ejected star, we use the rejection method to compute the ejection velocity distribution according to a Gaussian distribution with a $\mu = \overline{v}_{\text{ejec}}$ and $\sigma = 0.2 \cdot \overline{v}_{\text{ejec}}$. We eliminate from this catalog all stars with a $v_{\rm ejec}$ lower than the speed necessary to reach 10 kpc [3] in a given MW potential so that it can be observed or not within the range 3 - 5 $M_\odot$, which is the range probed by current surveys of HVSs.

An example distribution for a binary with $m_1 = 4\ M_\odot$, $m_2 = 0.5\ M_\odot$, $a_0 = 0.1$ AU, and $R_{\rm min} = 5$ AU can be seen in Figure 1. In this distribution, $\mu = \overline{v}_{\rm ejec} = 1850$ km s^{-1} and $\sigma = .2 \cdot \overline{v}_{\rm ejec}$ km s^{-1}.

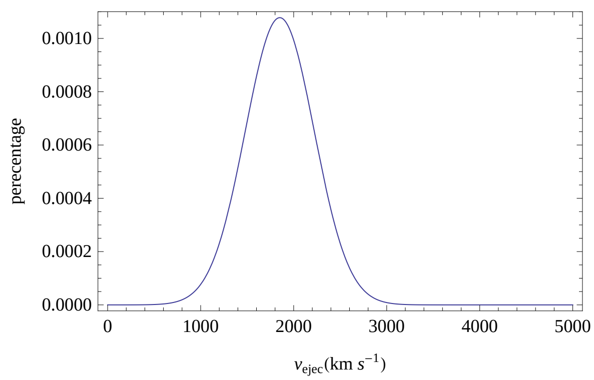


Figure 1: Sample Gaussian Distribution of $v_{\rm ejec}$ for $\overline{v}_{\rm ejec} = 1850$ km s^{-1} for $m_1 = 4\ M_\odot$, $m_2 = 0.5\ M_\odot$, $a_0 = 0.1$ AU, and $R_{\rm min} = 5$ AU

2.2 Orbit Integration in Distinct Milky Way Potentials

Determining the exact distribution of mass in the Milky Way (MW) potential is one of the great undertakings of modern astrophysicists. In this study, we attempt to account for and limit this uncertainty by integrating and comparing with observations the orbits of the synthetic HVSs using three models of the Milky Way potential (Bdef from Bromley et al. [1] and DB2d and DB4d from Dehnen and Binney [5]), all of which assume spherical symmetry in the Milky Way. We find each HVS's distance from the Galactic Center (GC) at a random time within its lifetime, only considering HVSs further than 10 kpc from the

[3] 1 kpc ≡ 1 kiloParsec = 206,264,806 AU

	ρ_0	r_c	Σ_0	M_b	r_b	M_d	R_d	M_h	r_h
Bdef	1.27×10^4	8	—	—	—	—	—	—	—
DB2d	—	—	53.3	8.19×10^9	1	5.0×10^{10}	8.5	57.5×10^{10}	21.8
DB4d	—	—	48.3	7.89×10^9	1	3.97×10^{10}	8.5	32.5×10^{10}	5.236
units	$M_\odot \mathrm{pc}^{-3}$	pc	$M_\odot \mathrm{pc}^{-2}$	$M_\odot$	kpc	$M_\odot$	kpc	$M_\odot$	kpc

Table 2: Parameters Used in MW Potential Models[1, 5]

GC and within 90 kpc for 3 $M_\odot$ stars, 120 kpc for 4 $M_\odot$ stars, and 150 kpc for 5 $M_\odot$ stars, distances suggested by current surveys of HVSs [3]. The distinct data sets that each of the three potentials yields allow us to determine which model of the MW potential is closer to observational reality.

We begin with a single-component model of the MW potential. This model was used in 2006 by Bromley et al. [1] and will be referred to as the Bdef model. The density equation used in this model is

$$\rho(r) = \frac{\rho_0}{1 + (r/r_c)^2}. \tag{8}$$

The Bdef model of the MW potential depends on two parameters: ρ_0, the central density, and r_c, the core radius, which are shown in Table 2 according to Bromley et al. [1].

The other two models of the MW potential, DB2d and DB4d, are found in a 1998 publication by Dehnen and Binney [5]. These models are both multi-component models which take into account the difference in the potentials of the galactic bulge (Φ_b), disc (Φ_d), and halo (Φ_h), which can allow us to determine that

$$\Phi_b(r) = \frac{-GM_b}{r_b + r}, \tag{9}$$

mass ($M_\odot$)	absolute magnitude	maximum $R_{\rm GC}$ (pc)
3	-0.3	144,544
4	-0.9	190,546
5.4	-2.4	380,189
15	-5	1,258,930

Table 3: Table of Stellar Magnitudes

$$\Phi_d(r) = -2\pi G \Sigma_0 R_d^2 \left(\frac{1 - e^{-r/R_d}}{r} \right), \tag{10}$$

and

$$\Phi_h(r) = \frac{-4\pi G M_h}{r} \ln \left(\frac{r + r_h}{r_h} \right). \tag{11}$$

The values of the parameters used in (9), (10) and (11) are defined in Table 2 according to the values mentioned by Dehnen and Binney [5].

We use as input to this simulation the values of mass, distance, time and velocity for each synthetic star as found in the simulation described in Section 2.1. Assuming that each star was ejected soon after its creation and that the time required for the star to reach $R_{\rm min}$ is also marginal, we find the position and velocity after a random time shorter than the lifetime of each star.

A star's absolute magnitude is a measure of its luminosity and thereby places a constraint on the distance at which the star can be observed from Earth. In this simulation, we reject all stars at a distance greater that that at which their absolute magnitude (or luminosity) would allow them to be observed by the current surveys. The HVS survey published by Brown et al. [3] suggests an apparent magnitude limit of 20.5. Table 3 contains the absolute magnitudes used for this rejection and the maximum distance at which a star of each magnitude can be observed.

	R_{GC} in kpc	v_{rf} in km s^{-1}
HVS1	111	696
HVS2	26	717
HVS3	62	548
HVS4	82	566
HVS5	45	649
HVS6	78	528
HVS7	60	416
HVS8	53	407
HVS9	68	485
HVS10	87	432
HVS11	70	336
HVS12	70	429
HVS13	125	443
HVS14	112	416
HVS15	85	343
HVS16	90	367

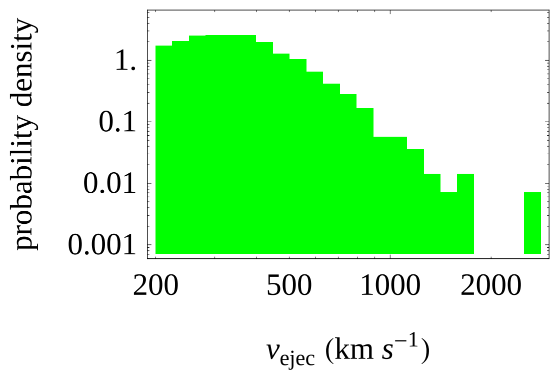
Table 4: Current Catalog of HVSs by Brown et al.(2009) [3]

3 Comparison With Observations

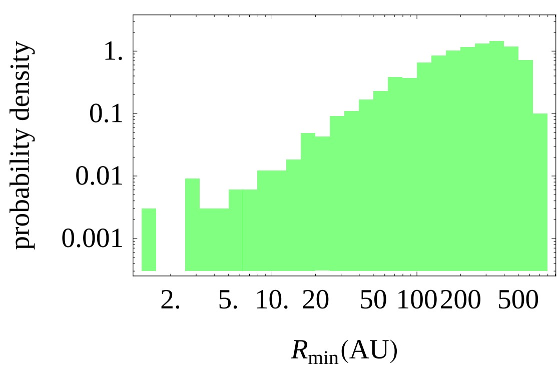
We compare the predicted distributions obtained in Section 2.1 and Section 2.2 with the most recent catalog of stars published by Brown et al. [3]. This catalog contains 16 HVSs, most of which are B-type stars found within a range of 25 kpc to 125 kpc from the GC. The rest frame velocities (v_{rf}) of the HVSs in the Milky Way potential range from 330 km s^{-1} to 720 km s^{-1}. The HVSs from this catalog, along with their distance from the Galactic Center (R_{GC}) and v_{rf}, are listed in Table 4.

In Figure 2, we plot the predicted distributions of R_{min} and v_{ejec} immediately following ejection. We use the 3289 stars generated in Section 2.1 and Section 2.2 to form these distributions.

After running these values in the orbit integrators for the three potentials, the number of HVSs that fit with the observed ranges is greatly reduced. The Bdef potential yields 2935 HVSs, the DB2d 1394 HVSs and the DB4d 753 HVSs. The differences in these data

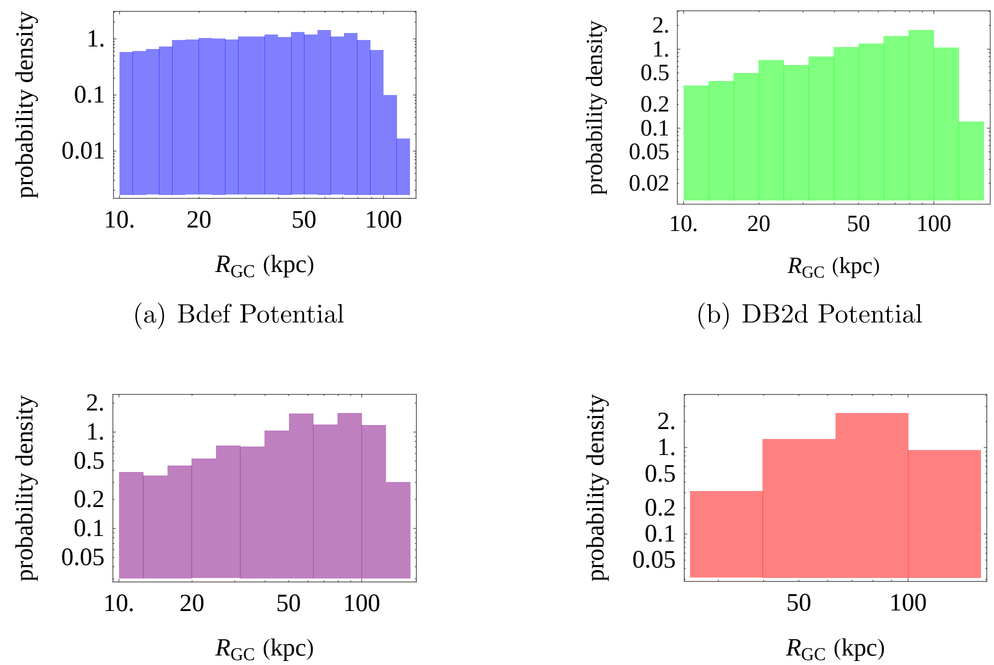


(a) $v_{\rm ejec}$



(b) $R_{\rm min}$

Figure 2: Ejection velocity, $v_{\rm ejec}$, and the minimum approach distance, $R_{\rm min}$, distributions



(a) Bdef Potential

(b) DB2d Potential

(c) DB4d Potential

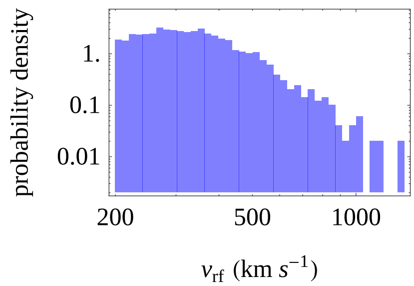
(d) Observations

Figure 3: Distributions of the distance from the GC, $R_{\rm GC}$, in three Milky Way potentials (a–c) and in the observations (d)

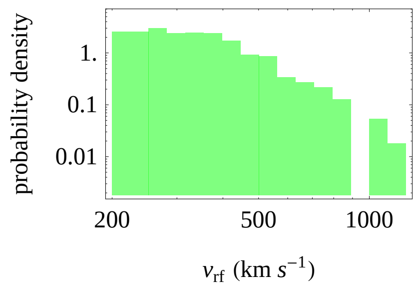
sets are an indicator of a bug in the code, which we are currently trying to eliminate. The distributions of $R_{\rm GC}$ and $v_{\rm rf}$ are plotted along with the observed distributions of these quantities in Figures 3 and 4 respectively. In Figure 5, we plot the relationship between $R_{\rm GC}$ and $v_{\rm rf}$.

4 Summary and Discussion

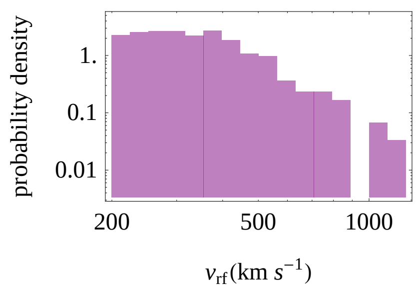
We used numerical simulations to predict the population of HVSs in the Milky Way Galaxy. We first generated a distribution of the $v_{\rm ejec}$ of such HVSs, assuming that they were produced in an exchange collision of a binary system with Sagittarius A*, the SMBH at the GC. Then, using the values generated in the first code, we integrated the orbits ($R_{\rm GC}$ and $v_{\rm rf}$) of the HVSs in three models of the Milky Way potential over a random time within their stellar



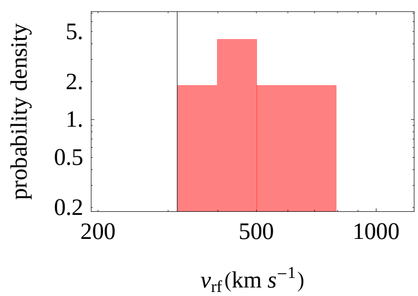
(a) Bdef Potential



(b) DB2d Potential



(c) DB4d Potential



(d) Observations

Figure 4: Distributions of the velocity, v_{rf}, of HVSs in three Milky Way potentials (a–c) and in the observations (d)

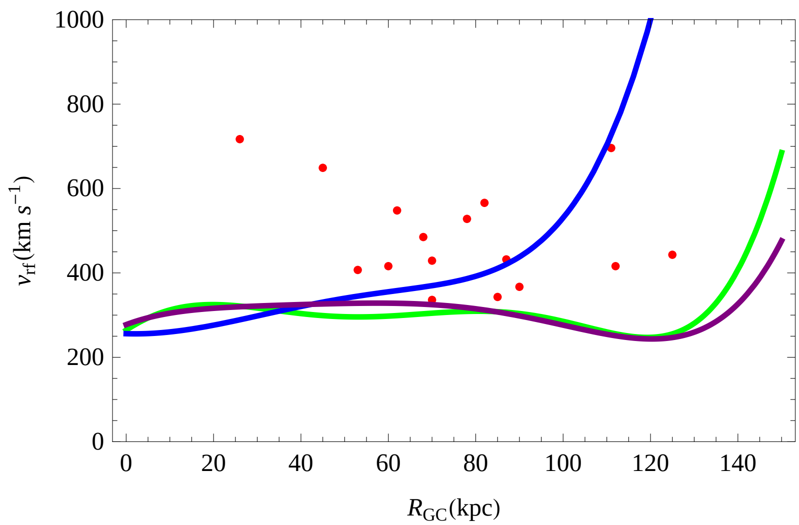


Figure 5: Distance from the GC, R_{GC}, and velocity, v_{rf}: The red points represent the observed stars while each line represents a different Milky Way potential. The colors are assigned as in Figure 3.

lifetime. We were then able to compare the catalog of synthetic stars created with the current catalog of 16 HVSs.

Due to the limited time available for this study, we are currently unable to place any constraints on the nature of the Milky Way Galaxy. We plan to improve the accuracy and scope of this work in the near future. We have identified a bug in the program used in this study, which may affect the results. Elimination of this bug would improve accuracy of the data. We can also optimize the program to allow it to run more quickly. Generation of a larger number of synthetic stars should provide sufficient data for an accurate statistical comparison, through use of a 2D Kolmogorov-Smirnov test, of the predicted distributions with the observations. We were unable to produce this large number of synthetic stars in this study due to time constraints, but it is possible that an improved statistical analysis will indicate constraints on the nature of the MW. In addition to the distributions for m_2 and a_0 used in this simulation, one could use a log-flat mass ratio to find m_2 and the distribution suggested by Duquennoy and Mayor [6] to find a_0. The use of different distributions of a_0 may also yield constraints on the nature of binaries. One could also investigate the effect of different ejection mechanisms as Sesana [13] did in 2007. The comparison in this study would become more accurate statistically if another, more comprehensive catalog of HVSs were to be released. Better galactic models, including those with non-spherical symmetry, may also improve the results of the orbit integrators.

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A Derivation of the Equation for the Average Ejection Velocity

To produce a distribution of the average ejection velocity of hypervelocity stars, we first derive an equation for the average ejection velocity. We begin by finding the tidal radius (R_{min}) of the binary system. At this location, the force of gravity that holds the binary system together is equal to the force of gravity on the star by the SMBH and

$$\frac{Gm_1m_2}{(a_0)^2} = \frac{Gm_1m_{\text{BH}}}{(R_{\text{min}})^2}. \tag{12}$$

Since the change in velocity of the star relative to the change in velocity of the binary causes the perturbation of the orbit, we differentiate both sides of (12) and find that

$$R_{\text{min}} = \sqrt[3]{\frac{m_{\text{BH}}}{(m_1+m_2)}}a_0. \tag{13}$$

We recall that energy is conserved in all closed systems, so we use the total energy of the binary system and, since that the total energy is negligible in comparison to the gravitational potential, we estimate the velocity of the star before its separation (v_0) as

$$v_0 \approx \sqrt{\frac{2Gm_{\text{BH}}}{R_{\text{min}}}}. \tag{14}$$

After tidal separation, the velocity of the star is changed by the amount $\delta v \ll v_0$, which is

$$\delta v = -\frac{1}{2}\sqrt{\frac{2M_{\text{BH}}G}{R^3_{\text{min}}}}, \tag{15}$$

and the ejection velocity ($v_{\rm ejec}$) is reached.

When the star is ejected, it uses the potential energy of the system to gain kinetic energy, as it becomes a hypervelocity star. The specific energy, which is the energy per unit mass, of a star after ejection ($E_{\rm ejec}$) is

$$E_{\rm ejec} = \frac{1}{2}{v_{\rm ejec}}^2. \tag{16}$$

To simplify (16), we eliminate all negligible values, substitute $E_{\rm ejec}$ for the expression of kinetic energy that it represents, and find that

$$\frac{1}{2}{v_{\rm ejec}}^2 \approx v_0 \delta v. \tag{17}$$

We substitute (14) into (17) to find an approximation for the average ejection velocity of an HVS:

$$v_{\rm ejec} \approx (1.49 \times 10^3 \text{km s}^{-1}) \left(\frac{m_{\rm BH}}{10^6 M_\odot}\right)^{\frac{1}{4}} \left(\frac{1 \text{ mpc}}{R_{\rm min}}\right)^{\frac{1}{4}} \left(\frac{\delta v}{383 \text{km s}^{-1}}\right)^{\frac{1}{2}}. \tag{18}$$