Increasing Efficiency though Flexibility in Airport Resource Allocation

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***Abstract*— When the scheduled demand at an airport exceeds the forecast capacity, the available capacity needs to be allocated among airlines in an efficient and equitable manner. The first stage of this two-step allocation process is a centralized optimization to determine the initial allocation among airlines. In the second stage, airlines cancel flights or modify their schedules based on flight-specific delay costs, which they hold private. The overall objective is to increase efficiency (that is, reduce total delay costs) to the extent possible.**

This paper demonstrates an inherent trade-off between the ability of the first-stage optimization to dynamically adapt to updates of the capacity forecast, and the flexibility available to airlines in the second stage. A new stochastic optimization model that balances this tradeoff is proposed. In addition to increasing the flexibility available to the airlines and the resultant delay reduction, the proposed formulation is shown to have attractive computational properties.

I. Introduction

Ground Delay Programs (GDPs) are used to mitigate airport congestion due to short-term capacity shortfalls by rationing the limited capacity among scheduled arrivals. The underlying principle of a GDP is that by delaying departure from its origin airport, a flight can avoid expensive airborne delays at a congested destination airport. More than 150,000 flights incurred 11 million minutes of GDP delays in 2013, making it one of the most frequently used air traffic management strategies [1].

The arrival capacity of an airport is measured by the number of landings that can be conducted in a certain amount of time, through discrete time-intervals known as slots. Given an arrival capacity forecast for some future time, the first step of a GDP is to allocate the available capacity among the scheduled arrivals. This step, known as the ground holding problem, aims to schedule the arrival times and slots (and consequently the departure times of flights at the origin airports) so as to minimize the sum of airborne and ground delay costs [2]. It is generally solved by assuming that the ground delay costs are identical for all flights, as are the airborne delay costs [3], and exempting flights of duration longer than a certain threshold (including international flights).

This assumption of identical delay cost functions across all flights is typically not true in practice, which necessitates the second step of a GDP. The slot allocations from the ground holding problem are revised by the airlines, based on their privately-held flight-specific delay costs. This step is conducted under a framework known as Collaborative Decision Making (CDM). The objective of the CDM revisions is to increase system efficiency by using airline information [4,5]. Apart from deciding which flights to cancel, airlines may improve their internal delay costs by reassigning flights among their own slots, and also exchange the slots corresponding to cancelled flights for alternate slots that can be used for later flights. This mechanism is preferable for the airlines since it does not require them to reveal their delay costs. The GDP framework is illustrated in Fig. 1.

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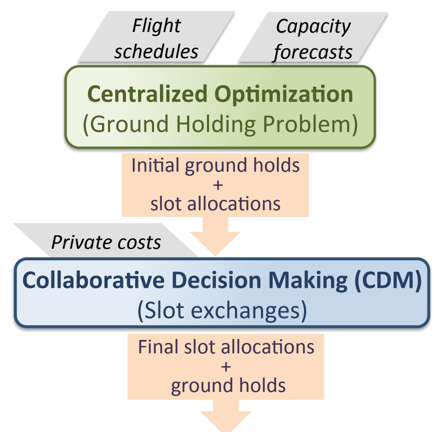

Fig. 1. Illustration of the overall GDP framework.

Airport capacity forecasts are subject to uncertainties, especially hours ahead of operation. Prior studies have therefore considered stochastic capacity forecasts comprising of multiple capacity scenarios and associated probabilities [6]–[10]. There are two broad classes of stochastic ground-holding models: Static stochastic models [7], which are single-stage integer stochastic programs that determine a single solution across all capacity scenarios, and dynamic stochastic models [9,11], which are multi-stage integer stochastic programs that allow for ground-hold solutions contingent on scenario materialization. Both models minimize the expected sum of ground and airborne delay costs. Under the assumption of identical flight delay costs, the dynamic stochastic model achieves lower delay costs. However, because the dynamic model differentiates flights based on their durations when determining their slot allocations, options for slot substitutions in the CDM step are more limited. This fact may result in higher final delay costs for the dynamic model compared to the static model, despite lower pre-CDM delay costs.

This paper combines the favorable features of the static and dynamic models to develop a *hybrid stochastic ground-holding model*. Similar to the dynamic model, the hybrid ground holding model uses the latest information on capacity scenario materialization in determining the ground-holding solution, but eliminates the dependence of its ground-holding solution on flight duration. As a result, the hybrid model provides more flexibility in the CDM step than the dynamic model. The paper also establishes two useful results on the tractability of the integer stochastic formulation of the hybrid model. The hybrid model is shown to consistently reduce total delay costs and improve efficiency (post-CDM) when the actual flight-specific delay costs are not homogeneous.

II. Stochastic ground-holding models

The inherent uncertainty in capacity predictions has motivated the development of stochastic formulations for the ground-holding problem.

A. Scenario trees

The most common representation of the uncertainty is in terms of scenarios, namely, alternate arrival capacity profiles for the airport, along with the associated probabilities. Such a forecast can be visualized in the form of a scenario tree, as shown in Fig. 2 for the case of a forecast with 5 possible scenarios. Scenario 1 (S1) corresponds to the airport starting

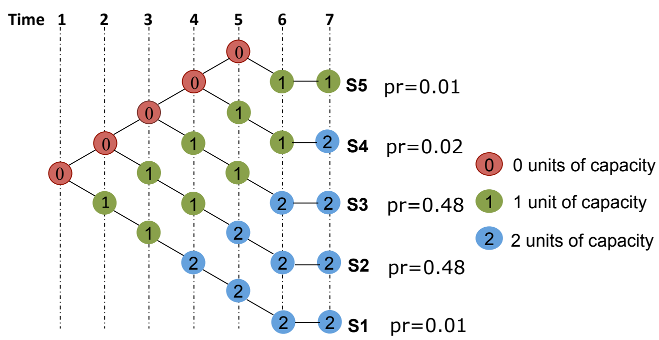

Fig. 2. Scenario tree example.

with zero capacity, improving to one unit of capacity in time-period 2, and further recovering to 2 units of capacity at time 4. Scenario S2 begins similarly with the airport at zero capacity, improves to one unit of capacity in time-period 3, and recovers to 2 units at time-period 5. The remaining scenarios follow, with the airport taking progressively longer to recover. At the start of the tree (time step 1), all scenarios are possible. But at time 2, either scenario S1 or scenarios S2-S5 are eliminated. The scenario tree also shows the a priori probabilities of occurrence of each scenario. The set of possible scenarios is denoted by Q, and the unconditional probability of scenario $q \in Q$ by π_q.

B. Static vs. dynamic ground holding models

As mentioned in Section I, a static model would determine a single ground-hold policy independent of which scenario materializes, whereas a dynamic model would determine scenario-dependent policies. Consider an example where an airport faces a capacity forecast from Fig. 2, and has two scheduled flights, F1 (with travel time of 2 time-steps) scheduled to arrive in time-period 3, and F2 (with travel time of 1 time-step) scheduled to arrive at time-period 4.

The solution to a static ground-hold policy [7] in this case would be to ground-hold F1 for 1 time-period, rescheduling it to leave at time-step 2 and arrive at time-step 4, and similarly rescheduling F2 to arrive at time-step 5. The total gate-hold time would be 2 time-periods, with an airborne delay of 2 units if S4 materializes (with probability 0.02), and 4 units if S5 materializes (with probability 0.01). If the homogeneous unit ground and airborne delay costs are assumed to be 0.5 and 2.5 respectively for both flights, the total cost of the static solution is $(2 \times 0.5) + 2.5 \times (0.04 + 0.04) = 1.2$.

By contrast, the dynamic ground-holding policy [9] would recommend that F1 receive 1 time-period of ground delay in all scenarios. F2 will not be delayed if S1 materializes, receive 1 time-period of ground-delay in either S2 and S3, 2 time-periods in S4 and 3 time-periods in S5, for an expected delay cost of 1.115.

Although the dynamic ground-holding solution has a lower delay cost than the static one, it comes at the expense of flexibility. Suppose, in the above example, that the airline operating F1 and F2 would like to swap their arrival slots, since the private flight-specific costs value F2 more than F1. Given the allocations from the static ground-holding problem, F2 will now depart at time-step 3 and arrive at time-step 4, while F1 will depart at time-step 3 and arrive at time-step 5. The slot allocations from the dynamic ground-holding problem are, however, scenario-dependent. On swapping with F2, F1 would have to depart at time-step 2 if S1 materializes, time-step 3 under S2 and S3, time-step 4 in S4 and time-step 5 under S5. This is not operationally feasible, since scenarios S3 and S4 are indistinguishable at time-step 3, when a decision would have to be made on whether F1 should depart. The nature of the dynamic ground-holding problem is such that CDM swaps are only possible between flights of equal duration, because the rules of CDM restrict airlines to use the same arrival slots assigned to them across different scenarios in the first GDP step. The benefit of slot swaps in CDM depends on the variability in the airline flight-specific delay costs: For example, if the ground and airborne delay costs of F2 are 9 times that of F1 (while maintaining the average delay costs assumed by the ground-hold problem), the total post-CDM delay cost of the static solution is 0.355, which is significantly lower than the optimal cost of 1.115 that could be achieved by the dynamic solution.

C. Hybrid stochastic ground-holding model

The observation that the multi-stage formulation of the dynamic model results in better pre-CDM allocations while the flight duration-independent allocations of the static model yields more slot swap options motivates the *hybrid stochastic ground-holding problem*:

$$\begin{aligned}
\text{Minimize} \quad & \sum_{q\in Q}\pi_q\left(\sum_{n=0}^{K}C_{g,n}\sum_{t=1}^{T-n}X^q_{t,t+n}+C_a\sum_{t=1}^{T}A^{\text{aq}}_{q,t}\right)\\
\text{subject to} \quad & \sum_{j=t}^{t+K}X^q_{t,j}=A^{\text{dem}}_t,\ \forall t\in\{1,..,T\},q\in Q \qquad (1)\\
& A^{\text{aq}}_{q,t}\geq\sum_{j=t-K}^{t}X^q_{j,t}+A^{\text{aq}}_{q,t-1}-A^{\text{cap}}_{q,t},\\
& \qquad \forall t\in\{1,..,T\},q\in Q \qquad (2)\\
& X^{q_1}_{t,j}=X^{q_2}_{t,j},\ \forall q_1,q_2\in G_{t-\text{max_dur}} \qquad (3)\\
& X^q_{t,j}\in\mathbb{Z}^+,\ \forall t,j\in\{1,..,T\},q\in Q
\end{aligned}$$

Notation:

T:	GDP planning horizon (input)
Q:	Set of possible scenarios, with a priori probabilities π_q (input)
$A^{\text{cap}}_{q,t}$:	Airport arrival capacity at time t under scenario q (input)
K:	Maximum number of time-steps for which any aircraft can be ground-held (input)
G_t:	Set of scenarios still possible at time t (input)
$C_{g,n}$:	Ground-delay cost incurred by an aircraft over n time-steps (input)
C_a:	Unit airborne delay cost (input)
A^{dem}_t:	Aggregate arrival demand at time t (input)
max_dur:	Duration of longest flight considered for rescheduling in the GDP (given)
$A^{\text{aq}}_{q,t}$:	Length of airborne arrival queue at time t for scenario q (decision variables)
$X^q_{t,t+n}$:	Number of flights rescheduled from arrival time t to arrival time $t+n$ for scenario q (decision variables)

The objective function minimizes the sum of expected ground and airborne delay costs. Constraint (1) ensure that no flight receives a ground delay of more than K time steps. Constraint (2) ensures that the number of aircraft that land at any time step does not exceed the airport arrival capacity under any realized scenario. Constraints (3) are coupling constraints (also known as non-anticipativity constraints) to ensure that the same ground-hold decisions are made for all scenarios that are indistinguishable at the time of a decision, in such a way that flights can be swapped irrespective of their duration. It is worth noting that the non-anticipativity constraints are a key difference between the static, hybrid and dynamic formulations: The static model would require that the same ground-hold decisions are made for all scenarios for all flights at all times; the dynamic model would require that the same ground-hold decisions are made for all scenarios that are indistinguishable at the time of a decision for all flights that have not yet departed; the hybrid model imposes this constraint in a way that allows swaps irrespective of the flight durations.

The proposed hybrid ground-holding problem formulation is a multi-stage stochastic mixed-integer program that permits scenario-specific revisions of the ground-holding solution like the dynamic model, and yet uses aggregate decision variables to avoid using individual flight durations (like a static model). At any time period t under capacity scenario q, the hybrid model assigns ground holds for all flights originally scheduled to land in time period $t+\text{max\_dur}$. This feature of the hybrid model ensures that slots assigned to flights of different durations can be swapped, since any flight of shorter duration than max_dur would be yet to depart when ground delays are assigned for flights arriving at $t+\text{max\_dur}$. When $\text{max\_dur}=T$, the hybrid formulation reduces to a single-stage stochastic program, namely, the static ground-holding model.

D. Benefits of the hybrid ground-holding model

1) CDM benefits: We revisit the example presented in Fig. 2 and Section II-B. Since the longest flight in the time-period of interest is F1, $\text{max\_dur}=2$. The optimal solution to the proposed hybrid ground-holding model would recommend that F1 receive 1 time-period of ground delay in all scenarios, departing at time-step 2 and arriving at time-step 4. F2 will not be delayed if S1 materializes, but receive 1 time-period of ground-delay in all other scenarios, for an expected delay cost of 1.195.

The optimal delay cost of the hybrid ground-holding model is less than that of the static model, but higher than that of the dynamic model. However, the slots assigned to flights F1 and F2 by the hybrid model can be swapped, if the airline so desires. F1 will depart in time-step 2 under scenario S1 and in time-step 3 for other scenarios, since S1 can be distinguished from the other scenarios by time-step 2, as seen in Figure 2. F2 will now depart at time-step 3, in all scenarios. The post-CDM expected delay cost in this case is 0.315. While this cost is an improvement over both the static and the dynamic allocations, one would expect that for typical delay cost variations between flights, the hybrid ground-hold model would result in a final delay cost between those of the static and dynamic models.

Table I summarizes the expected tradeoffs between the static, hybrid and dynamic stochastic ground-holding models, for typical cost structures.

	Static	**Hybrid**	**Dynamic**
Pre-CDM delay cost	High (Worst)	Medium	Low (Best)
Benefit from CDM	High (Best)	Medium	Low (Worst)
Equity	High (Best)	Medium	Low (Worst)
Tractability	High (Best)	Medium	Low (Worst)
Ease of implementation	High (Best)	Medium	Low (Worst)

TABLE I

TYPICAL TRADEOFFS EXPECTED IN THE STATIC, HYBRID AND DYNAMIC STOCHASTIC GROUND-HOLDING MODELS.

2) Equity: The static model is an aggregate model that does not differentiate between flights or scenarios, and yields a solution that maintains the original scheduled order of arrivals. The hybrid stochastic formulation may rearrange the original arrival schedule under some capacity scenarios, potentially resulting in inequities in the ground-hold allocation. Similar concerns have been raised about the dynamic ground-holding model [9]. The dynamic model typically delays short-haul arrivals under low capacity scenarios, as these arrivals are more responsive in the event of new forecast updates. By contrast, the hybrid ground-holding solution favors arrivals scheduled for later in the GDP, since they could potentially be advanced in the event of an early increase in airport capacity. Since the length of a GDP is not decided before the time of its initiation, the hybrid ground-holding solution is less prone to a systematic bias based on flight duration than the dynamic solution.

3) Computational tractability of the hybrid ground-holding model: This section outlines two results on the tractability of the hybrid stochastic model formulation under a fairly general set of conditions. These results assume integer demands ($A_t^d \in \mathbb{Z}^+,\ \forall t \in \{1,..,T\}$) and capacities ($A_{q,t}^{\text{cap}} \in \mathbb{Z}^+,\ \forall q \in Q,\ \forall t \in \{1,..,T\}$). The proofs use *perturbation analysis* to establish key properties of the optimal ground hold solutions that are sufficient conditions for the result statements. In this procedure, a possibly nonconforming optimal solution (i.e., an optimal solution that does not satisfy the key properties) is perturbed by an infinitesimal amount in the direction of a conforming solution, while ensuring no increase in the objective function value or violation of constraints. A feasible, conforming optimal solution is thereby constructed through a sequence of optimality-preserving perturbations to the nonconforming solution.

Case 1: Integer queue lengths

Lemma 1: The hybrid stochastic ground-holding formulation yields an optimal solution with integer values for all variables $X_{a,b}^q$ ($\forall q \in Q;\ a,b \in \{1,..,T\}$) if the queue length variables ($A_{q,t}^{\text{aq}}\ \forall q \in Q, t \in \{1,..,T\}$) are constrained to have integer values, and the ground-holding costs are marginally non-decreasing (i.e., $C_{g,n+1} - C_{g,n} \geq C_{g,n} - C_{g,n-1}\ \forall n$).

A sketch of the proof of Lemma 1 is presented in the appendix. The number of integer variables in the original hybrid ground-holding model is $O(T^3)$, since there were $O(T^3)$ ground-holding variables and $O(T^2)$ airborne queue lengths. Lemma 1 proves that an integral solution can be guaranteed by restricting the integrality requirement to only $O(T^2)$ variables.

Case 2: Capacity scenario tree with special structure

Capacity scenario trees often present a special structure with sequentially non-decreasing capacity scenarios. In other words, the sole element of uncertainty is the time at which the branches from the lowest capacity state to the next capacity state occur. An example of such a scenario tree with three capacity states (low (L), medium (M) and high (H), L<M<H) is shown in Figure 3.

Every capacity scenario in the tree in Figure 3 follows the same deterministic trend once the capacity transitions from state (L) to state (M). Regardless of the time step when capacity first increases from low state (L) to medium state (M), there are two successive medium capacity states (M) before the capacity rises to the high state (H). The duration of the lowest capacity (L) in scenario q is denoted $\text{dur}_q\ \forall q \in Q$.

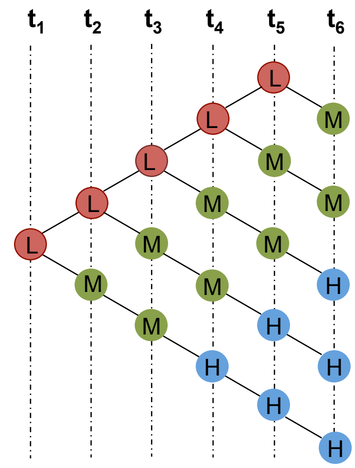

Fig. 3. Illustration of a capacity scenario tree with the special structure described in Case 2.

Given a scenario tree with this special structure, the scenarios can be labelled in increasing order of dur_q, i.e., $\text{dur}_q = q,\ \forall q \in \{1,..,|Q|\}$ and $|Q| = T$, without loss of generality. The lowest capacity state therefore lasts through the first time-step for scenario 1, and through the entire length of the GDP planning horizon (i.e, T intervals) for scenario T.

Lemma 2: Given marginally non-decreasing ground-holding cost coefficients $C_{g,n+1} - C_{g,n} \geq C_{g,n} - C_{g,n-1}, \forall n$, and a capacity scenario tree forecast with sequentially non-decreasing capacity scenarios and sole element of uncertainty being time of improvement from lowest capacity state, the hybrid ground-holding problem formulation is guaranteed to have an integral optimum solution if the queue length variables for scenario T (i.e., $A_{T,t}^{\text{aq}}\ \forall t \in \{1,..,T\}$) are constrained to be integers.

Therefore, total integrality under these conditions can be guaranteed by restricting the integrality requirement to $O(T)$ variables in the formulation, instead of $O(T^3)$ variables in the original formulation. The proof of Lemma 2 is also sketched in the appendix. The assumption of marginally non-decreasing ground holding costs is quite non-restrictive in practice, since each incremental delay only has an increased risk of propagating through the network.

III. Case studies

A comparative study of the static, hybrid and dynamic stochastic ground-holding models was conducted using data reported from a GDP at LaGuardia Airport (LGA) on Feb 17, 2006. The GDP was in effect from 7 am to midnight, during which there were 542 scheduled domestic arrivals, operated by 27 airlines. The original arrival schedule prior to the issuance of the GDP was obtained from the FAA's database [12]. Time-intervals were assumed to be 1 hr long, and flight durations were rounded up to the nearest hour.

The maximum flight duration among the domestic arrivals scheduled within the GDP time horizon was 5 hrs.

Representative estimates for unit ground and airborne delay costs for each flight were derived using T-100 schedules and P-52 data, capturing cost components like aircraft fuel and crew delays along with passenger delays [13]. The coefficient of variation in unit delay costs for the 10 airlines with the most flights ranged from 5% to 58%, with an average value of 19.4%. Future research would incorporate some of the cost functions determined in a recent paper by Bloem and Huang [14].

The arrival capacity of LGA is assumed to be 14 arrivals/15 min or 56 arrivals/hour [15]. Capacity scenario trees of the form shown in Figure 4 were assumed, with a reduced capacity of 28 arrivals/hour. The probability distributions for the scenario trees were randomly generated to obtain varying values of the expected duration of low-capacity, as well as the total length of the GDP.

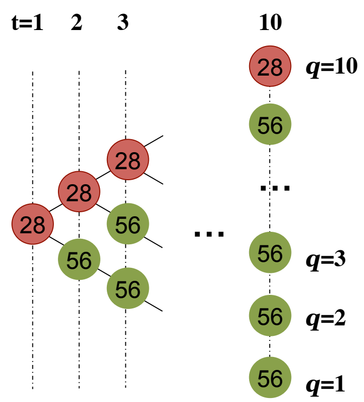

Fig. 4. Capacity scenario tree used in case studies.

The three stochastic ground-holding models were executed as the first GDP step under each of these distributions, followed by intra-airline slot substitutions for each airline. The run-times of the ground-holding optimization averaged under 1 second for the static and hybrid models, while the dynamic model typically took between 5-10 seconds. The scale of the scenario trees considered in the case studies, while resembling realistic conditions, is still limited. With increasing number of scenarios and capacity states, the discrepancy in the run-times between the models is expected to widen.

The slot substitution step was formulated as an assignment problem between flights and their corresponding set of scenario-specific slots, given the flight-specific delay costs. Feasibility of assigning a flight to a given set of scenario-specific slots is enforced based on its duration, as discussed earlier. Flight cancellations were not explicitly considered in this study.

The influence of the following GDP parameters in the context of CDM slot substitution is studied: [P1] Expected duration of reduced capacity, a measure of the level of severity of the GDP; and [P2] Total length of GDP planning horizon, a measure of the total length of disrupted operations.

Fig. 5 presents a comparison of the final delay costs aggregated across all airlines (after both the ground-holding allocation and the subsequent CDM slot swaps have been conducted) over a range of expected low-capacity durations, for GDP planning horizons of 7 hours and 15 hours. The figures present the net percentage improvement achieved by each model over the stochastic model with the largest final aggregated delay costs after intra-airline substitution. The missing bar in each set corresponds to the model with the highest delay costs in each case.

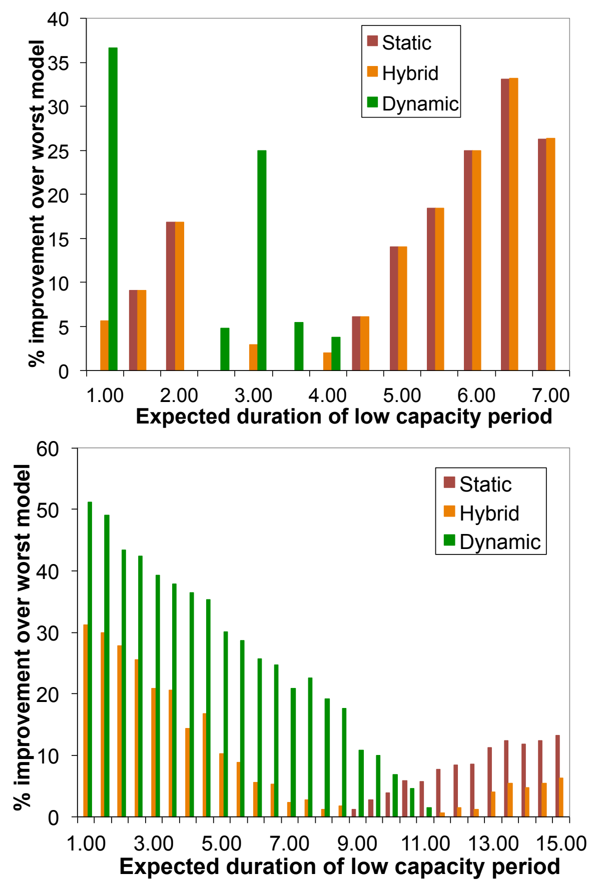

Fig. 5. Percentage improvement in final system delay costs for GDP horizon lengths of (top) 7 hrs and (bottom) 15 hrs, illustrating the impacts of the expected duration of reduced capacity and the total GDP planning horizon. The missing bars correspond to the formulation with the highest cost.

For a given GDP horizon length, the static model progressively takes over from the dynamic model as the model with the lowest final delay costs, as the expected duration of reduced capacity increases. The benefit of the dynamic model (for lower values of P1) increases with increasing GDP horizon length (P2). These trends are due to the greater pre-CDM benefits of the dynamic model in these regimes. The dynamic model performs better for longer GDP planning horizons (15 hrs vs. 7 hrs), because more dynamic information on capacities can be acquired and utilized.

The hybrid model is rarely the worst-performing model across the explored ranges of these test parameters. This observation validates the underlying principle of the hybrid model, which combines the superior pre-CDM performance of the dynamic model with the more flexible intra-airline substitution of the static model, to achieve consistent system delay cost reductions across a range of settings. While one might be led to believe that the greatest system benefits can be realized by choosing between the static or dynamic model depending on favorable GDP parameters, we note that the nature of slot allocation information exchanged between airport and airlines is specific to the selected model. There is merit to maintaining a consistent modeling framework across different GDP instances, for the sake of simplicity.

In addition to the total duration and level of severity of the GDP, the variability in unit delay costs (P3) is also an important factor. The greater the variability in the flight-specific delay costs, the more the likely benefits from CDM slot substitutions. As a result, different airlines can be impacted differently. More extensive simulations have shown that the static and hybrid models perform better than the dynamic model when there is a higher variability in unit delay costs. Similar benefits have also been demonstrated for mechanisms that allow slot exchanges between airlines [15].

A recently emerging family of ground-holding approaches look to eliminate the dependence of stochastic models on scenario tree-based inputs. Ball et al. [16] have proposed a Ration-by-Distance (RBD) model, which is an iterative, deterministic equivalent of a stochastic ground-holding problem. The RBD model relies on periodic capacity updates to perform corresponding adjustments to slot allocations (and the CDM-based airline responses).

Our work looks to address the GHP and CDM steps of a GDP in conjunction within a scenario-tree based architecture. A potential extension could consider a scenario-free framework involving capacity upgrades at various future points, wherein the GHP and CDM steps are revised at each upgrade. The performance of such a framework, which is closer to the state-of-practice, can be further compared to our current analysis to understand and quantify its merits.

IV. Conclusions

This paper proposed a new formulation for the stochastic single-airport ground holding problem with the objective of increasing the flexibility afforded to airlines in the Collaborative Decision Making stage of a Ground Delay Program, in order to increase the efficiency of the final allocation. The proposed hybrid stochastic ground-holding problem formulation combined the dynamic response of the dynamic ground-holding model with the ability to swap flights of different durations that was an attractive characteristic of the static ground-holding formulation. The paper showed that with marginally nondecreasing ground holding costs and when the only uncertainty in the capacity forecast was the time at which the capacity would improve, the hybrid ground-holding formulation was guaranteed to have an integer optimum solution if the arrival queue lengths in the most severe scenario were constrained to be integers.

The performance of the proposed hybrid model was evaluated using realistic data sets, both in terms of the initial ground-hold allocation, as well as the gains from the intra-airline substitution processes of the CDM framework. The detailed evaluation of the three models showed that the hybrid ground-holding model provided a more consistent benefit than the dynamic or static models in intra-airline substitution over a range of capacity uncertainties and variability in delay costs. A number of extensions are also possible. First, it is noted that the hybrid model strikes a balance between the static and dynamic model by aggregating flights over the maximum flight duration, as opposed to individual flight duration (dynamic model) or the entire planning horizon of the GDP (static model). The tradeoffs from this design can be studied by further varying the level of aggregation by only enabling swaps between flights of a certain length. Finally, the proposed formulations, while focused on the allocation of arrival resources, can be easily applied to the simultaneous allocation of arrival and departure resources at airports.

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Appendix: Proofs

Lemma 1. *The hybrid stochastic formulation yields an optimal solution with integer values for all variables $X^q_{a,b}$ ($\forall q \in Q$; $a, b \in \{1,..,T\}$) if the queue length variables ($A^{\text{aq}}_{q,t}$ $\forall q \in Q, t \in \{1,..,T\}$) are constrained to have integer values, and the ground-holding costs are marginally non-decreasing (i.e., $C_{g,n+1} - C_{g,n} \geq C_{g,n} - C_{g,n-1}$ $\forall n$).*

Proof: [Proof of Lemma 1] Assume an optimal solution X (with cost Z) for the hybrid problem satisfying the specified conditions on $C_{g,n}$, and with $A^{\text{aq}}_{q,t} \in \mathbb{Z}^+$, but with fractional values $X^q_{a,b}$ for some $q \in \{1,..,Q\}$; $a, b \in \{1,..,T\}$. X is converted into a fully integral solution through a sequence of perturbations that do not increase the optimal cost.

1) Amongst the fractional values, let $i = \min\limits_{a: X^q_{a,b} \notin \mathbb{Z}^+ \ \forall q \in Q} a$, and $j = \min\limits_{b: X^q_{i,b} \notin \mathbb{Z}^+ \ \forall q \in \{1,..,Q\}} b$. By manner of selection of indices i and j, there does not exist $p < i$ such that $X^q_{p,j} \notin \mathbb{Z}^+$ for any $q \in Q$.
2) Since $\sum\limits_{b=i}^{i+K} X^q_{i,b} = A^d_i \in \mathbb{Z}^+$, if $X^q_{i,j} \notin \mathbb{Z}^+$ and i from Step 1, there exists $b > j$ such that $X^q_{i,b} \notin \mathbb{Z}^+$. Let $k = \min\limits_{X^q_{i,b} \notin \mathbb{Z}^+ \text{and} X^q_{i,j} \notin \mathbb{Z}^+} b$.
3) The non-anticipativity constraints (3) imply that $X^q_{i,b}$ $\forall b \in i,..,i+K$ are the same for all $q \in G_{i-\text{max\_dur}}$. Let $\delta_{i,j} = 1 - \text{frac}(X^q_{i,j}), \delta'_{i,k} = \text{frac}(X^q_{i,k})$ for any $q \in G_{i-\text{max\_dur}}$.
4) Since $A^{\text{aq}}_{q,t} \in \mathbb{Z}^+$ $\forall q, t$, the scenario set $G_{i-\text{max\_dur}}$ can be partitioned into two non-overlapping subsets, Q^A_i and Q^B_i, such that:
   a) $\forall q' \in Q^A_i, \exists p_{q'}$ s.t. $i < p_{q'} < j$ and $X^{q'}_{p_{q'},j} \notin \mathbb{Z}^+$.
   b) $\forall q' \in Q^B_i$, no such time index $p_{q'}$ exists. Therefore, $\sum\limits_{l=j-K}^{j} X^{q'}_{l,j} \notin \mathbb{Z}^+$ (from (2) for time j and scenario q'). $A^{\text{aq}}_{q,j} \in \mathbb{Z}^+ \forall q \in Q \implies A^{\text{aq}}_{q',j} = 0$, implying that there is spare capacity, $A^{\text{cap\_rem}}_{q',j}$, at time j for scenario q' such that $A^{\text{cap\_rem}}_{q',j} = A^{\text{cap}}_{q',j} - \sum\limits_{l=j-K}^{j} X^{q'}_{l,j} \geq 1 - \text{frac}(X^{q'}_{i,j}) = \delta_{i,j}$.
5) $\forall q' \in Q^A_i$, time index $p'_{q'} = \min\limits_{i < p_{q'} < j, X^{q'}_{p_{q'},j} \notin \mathbb{Z}^+} p_{q'}$ is selected, and $\delta'_{p'_{q'},j} = \text{frac}(X^{q'}_{p'_{q'},j})$ computed.
6) Let $\delta_{\min} = \min(\delta_{i,j}, \delta'_{i,k}, \min\limits_{q' \in Q^A_i} \delta'_{p'_{q'},j})$.
7) For all $q \in G_{i-\text{max\_dur}}, q' \in Q^A_i$,

$$X\_\text{new}^q_{i,j} = X^q_{i,j} + \delta_{\min};\ X\_\text{new}^q_{i,k} = X^q_{i,k} - \delta_{\min};$$
$$X\_\text{new}^{q'}_{p'_{q'},j} = X^{q'}_{p'_{q'},j} - \delta_{\min};\ X\_\text{new}^{q'}_{p'_{q'},k} = X^{q'}_{p'_{q'},k} + \delta_{\min}$$

Feasibility of perturbed solution, X_new**:**
Case i. $\forall q' \in Q^A_i$: There was a balanced swap of $\delta_{\min}$ units of flow between times j and k, with no change to the reallocated arrival demand, $\sum\limits_{l=t-K}^{t} X^{q'}_{l,t}$, in the queue balance constraint (2) or airborne queue $A^{\text{aq}}_{q',t}$ $\forall t \in \{1,..,T\}$. Therefore, X_new is feasible.
Case ii. $\forall q' \in Q^B_i$: The perturbation transferred $\delta_{\min}$ units of flow from k to j, where $j < k$. Since there was spare capacity at time j for all $q' \in Q^B_i = A^{\text{cap\_rem}}_{q',j} \geq \delta_{i,j}$, a flow transfer of $\delta_{\min}$ can be fully absorbed with no additional airborne queue. In addition, the queue lengths at k and beyond ($A^{\text{aq}}_{q',t}$ $\forall t \geq k$) may be reduced by this transfer. Therefore, X_new is feasible.
Cost of perturbed solution: The airborne delay cost component for X_new cannot be greater than that of the original solution X. Therefore, the incremental ground delay cost for X_new compared to X (i.e., $Z_{\text{g}}(X\_\text{new}) - Z_{\text{g}}(X)$) is an upper bound on the total incremental cost $Z(X\_\text{new}) - Z(X)$. The incremental ground delay cost, $Z_g(X\_\text{new}) - Z_g(X)$ equals:
$$\begin{cases} (C_{g,j-i} - C_{g,k-i} + C_{g,k-p'_{q'}} - C_{g,j-p'_{q'}})\delta_{min}, \ \forall q' \in Q^A_i \\ (C_{g,j-i} - C_{g,k-i})\delta_{min}, \ \forall q' \in Q^B_i \\ 0, \ \forall q' \notin G_{i-\text{max\_dur}} \end{cases}$$
Since the ground-delay costs are marginally non-decreasing, and $i < p'_{q'} \leq j < k$, $C_{g,k-i} - C_{g,j-i} \geq C_{g,k-p'_{q'}} - C_{g,j-p'_{q'}} \geq 0$.
$$\implies Z(X\_\text{new}) - Z(X) \quad \leq \quad Z_g(X\_\text{new}) - Z_g(X) \leq 0,$$
implying that X_{new} preserves optimality.
8) Steps 1-7 are repeated until no fractional values remain in the optimal solution. The algorithm will terminate, since after every perturbation, no new fractional solution is created among $X^q_{a,b}$ $\forall a, b \in \{1,..,T\}, \forall q \in Q$, and at least one fractional solution is eliminated.

■

Lemma 2. *Given marginally non-decreasing ground-holding cost coefficients $C_{g,n+1} - C_{g,n} \geq C_{g,n} - C_{g,n-1}$, $\forall n$, and a capacity scenario tree forecast with sequentially non-decreasing capacity scenarios and sole element of uncertainty being time of improvement from lowest capacity state, the hybrid stochastic ground-holding formulation is guaranteed to have an integral optimum solution if the queue length variables for scenario T (i.e., $A^{\text{aq}}_{T,t}$ $\forall t \in \{1,..,T\}$) are constrained to be integers.*

Proof: [Proof of Lemma 2] The result is proved in two parts.
Part 1: Given certain conditions on the input parameters, the optimal solution is shown to have a special structure in terms of flight ordering with respect to the original schedule.
Part 2: For the given special structure of optimal solution, the stated result on the integrality of the optimal solution is proved.

In the discussion that follows, max_dur denotes the longest duration among all flights handled in the model, $A^{\text{cap\_rem}}_{q,b}(X)$ denotes the residual capacity for scenario q at time-step b given solution X, and $A^{\text{cap\_rem},a}_{q,b}(X)$ represents the residual capacity for scenario q at time-step b given solution X if only flows $X^q_{i,j}$ $\forall i \leq a-1, \forall j$ were considered. It is noted that $A^{\text{cap\_rem},T+1}_{q,b}(X) = A^{\text{cap\_rem}}_{q,b}(X)$.

Given the proposed scenario labeling scheme,

(A1) $G_q = \{q, q+1,..,|Q|\}$ $\forall q \in \{1,..,|Q|\}$ and $G_q \backslash G_{q+1} = \{q\}$ is a solitary scenario.
(A2) $A^{\text{cap}}_{q,t} \geq A^{\text{cap}}_{q+1,t}$ $\forall q \in \{1,..,|Q|\}$, $\forall t \in \{1,..,T\}$.

The special scenario tree structure enables a compact representation of the hybrid ground-holding solution. A partial solution $X^q(i:j)$ is the ground-holding allocation under scenario q for flights scheduled to arrive between time-steps i and j. By the principle of the hybrid ground-holding model, flights scheduled to arrive at time-step $t + \text{max\_dur}$ are assigned ground delays at time-step t, based on observed capacities up to time t. Accounting for non-anticipativity constraints, the ground-hold solution for a given scenario q can be expressed as the union of two partial solutions: $X^q(1:T) = [X^T(1:q+\text{max\_dur}) \quad X^q(q+\text{max\_dur}+1:T)]$. The component $X^T(1:q+\text{max\_dur})$ captures the ground-hold decisions taken up to time-step q and is common to all scenarios indistinguishable until this time-step (i.e., G_q). The subsequent decisions from time-step $q+1$ onwards ($X^q(q+\text{max\_dur}+1:T)$) are taken independently for scenario q, following its divergence from scenario cluster G_q.

At time-step t, two sub-problems that determine partial ground-holding solutions $X^q(t+\text{max\_dur}:T)$ $\forall q \in G_t$ and $X^{t-1}(t+\text{max\_dur}:T)$, given partial solution $X^T(1:t+\text{max\_dur}-1)$, are considered: The first subproblem corresponds to a deterministic ground-holding problem that is solved for the branch of the scenario tree that becomes certain at time t, while the second one corresponds to the portion of the scenario tree that is still uncertain at time t. Fig. 6 illustrates the two sub-problems at time t, namely, the deterministic (in green) and the stochastic (in red).

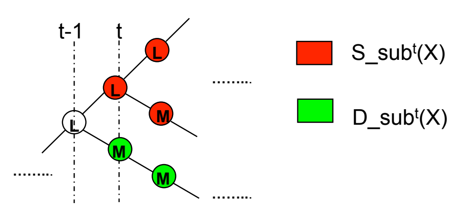

Fig. 6. Illustration of sub-problems at time t for given scenario tree.

Deterministic ground-holding subproblem ($\mathbf{D\_sub^t(X_0)}$): Given a partial ground-holding solution, $X^T_0(1:t+\text{max\_dur}-1)$, the partial solution $X^{t-1}_0(t+\text{max\_dur}:T)$ is the solution to the following problem:

$$\min \sum_{n=0}^{K} C_{g,n}(\sum_{i=t+\mathrm{max\_dur}}^{T-n} X^q_{i,i+n}) + C_a \sum_{i=t+\mathrm{max\_dur}}^{T} A_i^{\mathrm{aq,D\_sub}^t}$$
$$\text{s.t.} \quad \textstyle\sum_{j=i}^{i+K} X^q_{i,j} = A^d_i,\ \forall i \in \{t+\mathrm{max\_dur},..,T\}$$
$$A_i^{\mathrm{aq,D\_sub}^t} \geq \textstyle\sum_{j_0}^{i} X^q_{j,i} + A_{i-1}^{\mathrm{aq,D\_sub}^t} - A_{q,i}^{\mathrm{cap\_rem},t+\mathrm{max\_dur}}(X_0),$$
$$j_0 = \max(t+\mathrm{max\_dur}, i-K); \forall i \in \{t+\mathrm{max\_dur},..,T\}$$
$$X^q_{i,j} \in \mathbb{Z}^+,\ \forall i,j \in \{t+\mathrm{max\_dur},..,T\}$$

The arrival queue length in this subproblem for time interval i is denoted as $A_i^{\mathrm{aq,D\_sub}^t}$. Due to the inherent property of deterministic ground-holding, flight ordering in partial solution $X_0^{t-1}(t+\mathrm{max\_dur}:T)$ for scenario $t-1$ in any solution X_0 will be the same as in the original schedule.

Stochastic ground-holding subproblem ($\mathbf{S\_sub^t(X_0)}$): Given a partial solution $X_0^T(1:t+\mathrm{max\_dur}-1)$, the partial solutions $X_0^q(t+\mathrm{max\_dur}:T)\ \forall q \in G_t$ are given by the solution to the following stochastic ground-holding problem, that minimizes the expected delay cost subject to:

$$\sum_{j=i}^{i+K} X^q_{i,j} = A^d_i,\ \forall i \in \{t+\mathrm{max\_dur},..,T\}, q \in G_t$$
$$A_{q,i}^{\mathrm{aq,S\_sub}^t} \geq \sum_{j=j_0}^{i} X^q_{j,i} + A_{q,i-1}^{\mathrm{aq,S\_sub}^t} - A_{q,i}^{\mathrm{cap\_rem},t+\mathrm{max\_dur}}(X_0),$$
$$j_0 = \max(t+\mathrm{max\_dur}, i-K),\ \forall i \in \{t+\mathrm{max\_dur},..,T\},\ q \in G_t$$
$$X^{q_1}_{i,j} = X^{q_2}_{i,j},\ \forall q_1, q_2 \in G_{i-\mathrm{max\_dur}}, \forall i \in \{t+\mathrm{max\_dur},..,T\}$$
$$X^q_{i,j} \in \mathbb{Z}^+,\ \forall i,j \in \{t+\mathrm{max\_dur},..,T\}$$

The arrival queue length at time i in this subproblem is denoted $A_{q,i}^{\mathrm{aq,S\_sub}^t}, \forall q \in G_t$.

Lemma 2, Part 1: It is first shown that there exists an optimal solution X for the hybrid ground-holding formulation such that the ground-holding allocation for scenario T, $X^T(1:T)$, has the same ordering of flights as in the original schedule. The proof is based on perturbation analysis (similar to Lemma 1), and is omitted here in the interest of space. Details can be found in [15].

Proof of Lemma 2, Part 2 We now have to prove that an optimal solution X is integral under the additional condition that $A^{\mathrm{aq}}_{T,t} \in \mathbb{Z}^+\ \forall t \in \{1,..,T\}$.

The result in Part 1 concerning the structure for the optimal solution X holds for any general value of $A^{\mathrm{aq}}_{s,t}\ \forall s\ \forall t$. Therefore, the structure holds true for specific case of integral $A^{\mathrm{aq}}_{T,t},\ \forall t$. Once again, we adopt a perturbation analysis for this proof.

1) **Non-conforming solution:** Assume we have a non-integral optimal solution X such that for scenario T (longest duration of lowest capacity state) there exist time instances $p \in \{1,..T\}$, $j \in \{p, p+1, .. \min(p+K,T)\}$ such that $X^T_{p,j} \notin \mathbb{Z}^+$. Let us select the earliest such time instance p, and corresponding earliest time instance j for which $X^T_{p,j} \notin \mathbb{Z}^+$. In accordance to the structure for X as derived in Part 1, the ordering for flights in X for scenario T is the same as in original schedule.
Since, for a given p, j is the lowest time index for which $X^T_{p,j} \notin \mathbb{Z}^+$, $\exists$ time instance $q > j$ s.t. $X^T_{p,q} > 0$, since $\sum_{t=p}^{\min(p+K,T)} X^T_{p,t} = A^d_p$ where $A^d_p \in \mathbb{Z}^+$. Let q be the lowest such time instance. Given the order preserving structure of the solution for scenario T, we can infer that no arrival originally scheduled beyond time p is allotted to any time at or before j. i.e. $X^T_{k,t} = 0\ \forall k > p, t \leq j$. Also, since p is the lowest time index for which $X^T_{p,j} \notin \mathbb{Z}^+$, we have $X^T_{k,l} \in \mathbb{Z}^+\ \forall k < p, l \in \{k, k+1, .. \min(k+K, j)\}$. Therefore, $\sum_{t=\max(1,j-K)}^{j} (X^T_{t,j}) = \sum_{t=\max(1,j-K)}^{p-1} (X^T_{t,j}) + X^T_{p,j} \notin \mathbb{Z}^+$, since $\sum_{t=\max(1,j-K)}^{p-1} (X^T_{t,j}) \in \mathbb{Z}^+$.
We know that $A^{\mathrm{aq}}_{T,j} = \min(0, \sum_{t=\max(1,j-K)}^{j} X^T_{t,j} + A^{\mathrm{aq}}_{T,j-1} - A^{\mathrm{cap}}_{T,j})$, where $A^{\mathrm{cap}}_{T,j}$ and $A^{\mathrm{aq}}_{T,j-1} \in \mathbb{Z}^+$. Therefore, if $A^{\mathrm{aq}}_{T,j} \in \mathbb{Z}^+$, the only possibility is that $A^{\mathrm{aq}}_{T,j} = 0$. This implies that capacity is not exceeded at time j for scenario T, that is, $A^{\mathrm{cap\_rem},p+1}_{T,j}(X) = A^{\mathrm{cap}}_{T,j} - A^{\mathrm{aq}}_{T,j-1} - \sum_{t=\max(1,j-K)}^{j} (X^T_{t,j}) \geq 0$ and $\notin \mathbb{Z}^+$.
The above, in turn, implies $A^{\mathrm{cap\_rem},p+1}_{T,j}(X) > 0$. From property (A2) of scenario tree, we can conclude that $A^{\mathrm{cap\_rem},p+1}_{q,j} > 0\ \forall s \in \{1,..,T\}$.
As per the hybrid stochastic model's working principle, the ground-holding decision $X^T_{p,j}$ is taken at time $p - \mathrm{max\_dur}$, and affects scenarios in set $G(p - \mathrm{max\_dur})$. There are three possible categories of scenarios within $G_{p-\mathrm{max\_dur}}$:
   - Type 1. $q \in G_{p-\mathrm{max\_dur}}$ such that $A^{\mathrm{cap\_rem}}_{q,j}(X) > 0$
   - Type 2. $q \in G_{p-\mathrm{max\_dur}}$ such that $A^{\mathrm{cap\_rem}}_{q,j}(X) = 0$, but $\exists m$ such that $p \leq m < j$ and $X^s_{m,j} > 0$.
   - Type 3. $q \in G_{p-\mathrm{max\_dur}}$ such that $A^{\mathrm{cap\_rem}}_{q,j}(X) = 0$, and $\nexists m$ such that $p \leq m < j$ and $X^q_{m,j} > 0$

   Note that scenario T falls into Type 1.
2) **Perturbation:** Consider a new solution X_new obtained by advancing δ units of ground-hold allocation $X^T_{p,r}$ to $X^T_{p,j}$ as follows:
$$X\_\mathrm{new}^T_{p,j} \quad = \quad X^T_{p,j} + \delta;\ X\_\mathrm{new}^T_{p,r} = X^T_{p,r} - \delta, \qquad (4)$$
all else being equal. For scenario $s \in G(p - \mathrm{max\_dur})$ belonging to Type 2, we consider additional perturbation involving a balancing transfer of δ units from $X^q_{m,j}$ to $X^q_{m,r}$.
$$X\_\mathrm{new}^q_{m,j} \quad = \quad X^q_{m,j} - \delta;\ X\_\mathrm{new}^q_{m,r} = X^q_{m,r} + \delta, \qquad (5)$$
all else being equal.
3) **Feasibility of perturbation:** Given the perturbation is essentially a rearrangement of ground-holding allocation for arrivals scheduled for time indices p under scenario T (and m for scenario q belonging to Type 2), its feasibility is not affected in any way.
4) **Cost of perturbation:** We now consider cost of perturbation specific to scenarios from each of the above three categories.
   - Type 1. The unbalanced δ units of ground-hold re-allocation to time index l (from $X^T_{p,r}$) are absorbed by the available spare capacity (since $A^{\mathrm{cap\_rem}}_{p-\mathrm{max\_dur},j}(X) > 0$) without producing any queue. We can thereby show highest cost of perturbation = $Z_q(X\_\mathrm{new}) - Z_q(X) = (C_{g,j-p} - C_{g,r-p})\delta < 0$ (corresponds to situation where the perturbation causes no decrease to airborne delay costs).
   - Type 2. We can show that cost of perturbation = $Z_q(X\_\mathrm{new}) - Z_q(X) = (C_{g,j-p} - C_{g,r-p} + C_{g,r-m} - C_{g,j-m})\delta \leq 0$ for marginally non-decreasing ground delay costs.
   - Type 3. We can show that $Z_q(X\_\mathrm{new}) - Z_q(X) = (C_{g,j-p} - C_{g,r-p} + C_a(r-j))\delta \leq 0$.

   Therefore, Z(X_new)≤Z(X).

We can repeat the above-described perturbations until we have an optimal solution that bears only integral values for $X^T_{p,j}\ \forall p \in \{1,..,T\}, j \in \{p, p+1, .., \min(p+K,T)\}$.

As shown earlier, the compact representation for ground-holding solution for any scenario $q \in \{1,..,|Q|\}$ is $X^q(1:T) = [X^T(1:q+\mathrm{max\_dur})\quad X^q(q+\mathrm{max\_dur}+1:T)]$, where the partial solution $X^q(q+\mathrm{max\_dur}+1:T)$ can be obtained as solution to the deterministic sub-problem $\mathrm{D\_sub}^{q+1}(X)$.
Given integral values for $X^T(1 : T)$, we know that $A^{\mathrm{cap\_rem},q+1+\mathrm{max\_dur}}_{q,t}(X) \in \mathbb{Z}^+\ \forall t \geq q+1+\mathrm{max\_dur}$. Therefore, the solution to the deterministic sub-problem $\mathrm{D\_sub}^{q+1}(X)$ will also be integral for all q, ensuring that the overall ground-holding solution $X^q(1:T)$ will be integral for all q.

In summary, if the queue length variables for scenario T (longest duration of lowest capacity state) are restricted to be integral, the hybrid stochastic ground-holding model will yield an integral optimum under (1) marginally non-decreasing ground-holding cost coefficients, and (2) capacity scenario tree with sequentially non-decreasing capacity scenarios, with the sole element of uncertainty being the time of improvement from the lowest capacity state. ■