

# A Queuing Model of the Airport Departure Process

Ioannis Simaiakis and Hamsa Balakrishnan

Massachusetts Institute of Technology  
{ioa\_sim@mit.edu, hamsa@mit.edu}

This paper presents an analytical model of the aircraft departure process at an airport. The modeling procedure includes the estimation of unimpeded taxi-out time distributions, and the development of a queuing model of the departure runway system based on the transient analysis of  $D(t)/E_k(t)/1$  queuing systems. The parameters of the runway service process are estimated using operational data. Using the aircraft pushback schedule as input, the model predicts the expected runway schedule and takeoff times. It also estimates the expected taxi-out time, queuing delay and its variance for each flight, in addition to the congestion level of the airport, sizes of the departure runway queues and the departure throughput. The proposed approach is illustrated using a case study based on Newark Liberty International (EWR) Airport. The model is trained using data from 2011, and is subsequently used to predict taxi-out times in 2007 and 2010. The predictions are compared with actual data to demonstrate the predictive capabilities of the model.

*Key words:* Airport operations; Aircraft departure processes; Taxi-out times; Surface congestion

*History:*

---

## 1. Introduction

Airport surface congestion has several undesirable impacts, the most noticeable of which is an increase in taxi-out times. An analysis of operations in the year 2007 at John F. Kennedy (JFK), Newark Liberty (EWR) and Philadelphia (PHL) airports showed that they were congested 10-20% of the time (Simaiakis and Balakrishnan 2010). The study also showed that departures operating when EWR was congested had an average taxi-out time of 52 min, significantly greater than their average unimpeded (ideal) taxi-out time of 14 min.

The first step to mitigating the impacts of surface congestion is characterizing the relationship between airport congestion and taxi-out delays. Understanding this relationship would help us develop congestion control mechanisms that will ultimately reduce excessive taxi-out times and the associated costs, including fuel burn, emissions and controller workload. This paper develops a generalizable and adaptable model of the departure process. The modeling process is illustrated using a case study of EWR operations. The model, which is calibrated using data from 2011, is used to predict taxi-out times and surface congestion for departures at the two main runway configurations of EWR in the years 2007 and 2010. The model is shown to also be useful for

predicting aggregate taxi-out times and departure queues over a few hours or a day, as well as the taxi-out times of individual flights.

It is important to note that the impact of uncertainty in the pushback schedules is not investigated in this work. In other words, the predictive properties of the proposed models are studied assuming that the pushback schedules are known. In the current system this may only be realistic for short time horizons (of about 15 minutes).

## 2. Related work

Prior work on the modeling of the departure process at airports can be broadly classified into three groups. The first group focuses on computing airport-related delays with analytical queueing models, to enable fast-time simulations of the National Airspace System (NAS). In particular, Kivestu (1976), Malone (1995), Pyrgiotis et al. (2011) focus on runway-related delays. This runway-centric approach is justified by the observation that the runway system is the main throughput bottleneck at an airport (Idris et al. 1998). These models are useful for long-term planning (for example, estimating the expected reduction in delays from the construction of a new runway), or for estimating the network propagation of local disruptions, such as thunderstorms. However, they are less effective for studying congestion management algorithms, or for predicting taxi-out times for individual flights. In addition, these models use a Poisson process for modeling the aircraft service requests at the runway server, an assumption that has only been validated for aircraft arrivals (Willemain et al. 2004), and not for departures. Jacquillat and Odoni (2012) has recently suggested that simulating service requests with a less random process (one with smaller support and variation around the expected value) better predicts both the expected delays and their variability at an airport. Other analytical queueing models of the NAS have tried to incorporate ground operations (Clarke et al. 2007, Long et al. 1999, Wieland 1997). In particular, the Detailed Policy Assessment Tool (DPAT) was the first system-wide model to include delays not related to the runway queueing, using a user-defined stochastic input for taxiway-related delays in addition to a deterministic runway module (Wieland 1997). By contrast, LMINET, a queueing network model of the NAS developed by the Logistics Management Institute, modeled taxiway delays using an M/M/1 queue (Long et al. 1999). It is worth noting that the airport modules in the above models have not been fully validated.

The second category of prior research has focused on predicting taxi-out times by creating “mesoscopic” simulations of the departure process. Shumsky (1995, 1997) developed a model to predict taxi times using a variety of explanatory variables such as the airline, the departure runway

and departure demand and a queuing model for the runway service process. However, the queuing model was based on cumulative behavior and did not reflect the stochastic nature of the process. Idris et al. (2001), as part of the Departure Planner project (Feron et al. 1997), analyzed the main causal factors that affect taxi times and developed a statistical regression model to predict them. They did not explicitly model the runway service process, and so could not link the excessive taxi times with the capacity constraints. Their model could therefore not be used for strategic surface flow management, which requires accounting for gate-to-runway traffic levels, and the better regulation of surface queues. Motivated by the above observations, Pujet et al. (1999) developed a simple stochastic queuing model to predict taxi times. They assumed that an aircraft needs a certain (fixed) amount of time, defined to be the *travel time*, to reach the departure runways. In their model, upon reaching the departure runways, aircraft line up in the *runway queue*, where they get served by the runway server according to a probabilistic service process. They estimated the travel time for each flight, and modeled the probabilistic service process. Carr (2004) extended the mesoscopic models by creating a complete set of modules that could cause departure delays (arrivals, departure sequencing, runway crossings, departure fixes constraints), while Simaiakis and Balakrishnan (2009) provided a more complete queuing model of the departure process, by using better unimpeded taxi-out time estimations, new models for the runway server(s) and the ramp and taxiway delays. However, none of the above surface models account for the impact of arrivals nor air traffic flow management programs, although they are known to significantly impact taxi-out times (Idris 2001). In addition, these models require multiple runs in order to yield statistically significant results.

A third body of related work involves the microscopic modeling of all airport components, such as the Airport and Airspace Simulation Model (SIMMOD) and the Total Airspace and Airport Modeler (TAAM). These tools model the layout of an airport, the operating rules for every aircraft type, and the dynamics of every gate, taxiway and runway with high fidelity. They need extensive adaptation, and repeated, computationally-intensive simulations to generate statistically significant results (Odoni et al. 1997).

### 3. Model inputs and outputs

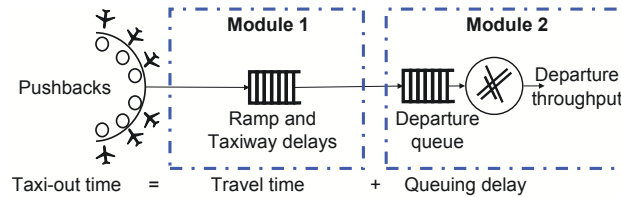
This paper focuses on predicting airport performance assuming that the pushback schedules are known, and does not investigate the impact of uncertainty in the pushback schedules. The inputs to the proposed model are: (1) Pushback times of departing flights; (2) Arrival throughput during the 15-minute period starting at time  $t$ , denoted  $A(t)$ ; (3) Segment in use,  $(MC; RC)$ , namely, the

visibility conditions,  $MC$ , and the runway configuration,  $RC$ ; and, if possible, (4) Airspace route availability in the 15-minute period starting at time  $t$ , denoted  $S_{RAPT}(t)$ . Data from the Route Availability Planning Tool (DeLaura and Allan 2003) is averaged over 13 departure routes in the New York area to obtain a metric of route availability:  $S_{RAPT}(t) = 0$  denotes no route blockage at all during the entire time period, whereas  $S_{RAPT}(t) = 3$  would imply that all routes were severely blocked during the entire time period.

The outputs of the model include: (1) Number of takeoffs in the 15-minute period starting at time  $t$ , denoted  $T(t)$ ; (2) Total number of aircraft taxiing out (indicator of surface congestion level) at the start of period  $t$ , denoted  $N(t)$ ; (3) Number of aircraft waiting in the departure queue (the physical queue at the thresholds of the departure runways) at the beginning of period  $t$ , denoted  $Q(t)$ ; (4) Number of departing aircraft *traveling* in the ramps and the taxiways towards the departure queue at the beginning of period  $t$ , denoted  $R(t)$ ; (5) Expected taxi-out time of departing aircraft  $l$ ,  $\mathbb{E}[\tau(l)]$ ; (6) Expected queuing delay at the departure queue for aircraft  $l$ ,  $\mathbb{E}[D_l]$  and its variance, denoted  $\text{var}(D_l)$ ; (7) Number of aircraft taking off between the pushback and takeoff time of aircraft  $l$  (known as the length of the takeoff queue experienced by aircraft  $l$  (Idris et al. 2001)), denoted  $N_Q(l)$ ; and (8) Runway schedule (times at which aircraft arrive at the departure queue).

#### 4. Model structure

The proposed model, shown in Figure 1, consists of two modules, namely, the process between aircraft pushing back from their gates and traveling to the runway, and the queuing process at the departure runway.



**Figure 1** Departure process model.

By modeling the departure process in this manner, the taxi-out time  $\tau_l$  of each departing aircraft  $l$  can be expressed as

$$\tau_l = \tau_l^{\text{travel}} + D_l \quad (1)$$

where  $\tau_l^{\text{travel}}$  is the travel time of each departing aircraft  $l$  from its gate to the departure runway, and  $D_l$  is the queuing delay that aircraft  $l$  experiences. The link between the two modules is through the output of Module 1, namely, the runway schedule.

In the context of this model, a pushback is defined as the moment in time when the aircraft is physically detached from its gate and the taxi-out time is defined as the time between the pushback and the takeoff.

#### 4.1. Data sources

The main source of data used in this paper is the Aviation System Performance Metrics (ASPM) database, which covers 77 major airports in the United States (FAA Accessed February 2012). The ASPM database contains the actual pushback time and actual takeoff time for every flight in the database. It also reports the runway configuration and the local meteorological conditions at each airport.

### 5. Travel time estimation (Module 1)

The first module of the model calculates the travel time of aircraft from their gates to their departure runway (Figure 1). The associated process can be conceptually described as follows: Aircraft pushback from their gates according to the pushback schedule. They enter the ramp and then the taxiway system, and taxi to the departure queue which is formed at the threshold of the departure runway. During this traveling phase, aircraft interact with each other. For example, aircraft queue to get access to a confined part of the ramp, to cross an active runway, or to enter a taxiway segment on which another aircraft is taxiing, or they get rerouted to avoid passing through already congested spots. These spatially distributed queues are cumulatively denoted as *ramp and taxiway interactions*, and the associated delays are represented by an additive term,  $\tau_{\text{taxiway}}$ . The travel time  $\tau_{\text{travel}}$  of each departing aircraft is therefore expressed as:

$$\tau_{\text{travel}} = \tau_{\text{unimped}} + \tau_{\text{taxiway}} \quad (2)$$

where the first term of Equation (2),  $\tau_{\text{unimped}}$ , reflects the nominal or *unimpeded taxi-out time* of the flight. This is the time that the aircraft would spend in the departure process if it were the only aircraft on the ground. The second term,  $\tau_{\text{taxiway}}$ , reflects the delay due to ramp and taxiway interactions with other aircraft that are also on their way to the departure queue. The number of such aircraft is given by  $R(t) = N(t) - Q(t)$ . The magnitude of this delay will depend on the exact nature of interactions among the taxiing aircraft, that is, the level and location of congestion in the ramp and the taxiways.

#### 5.1. Unimpeded taxi-out time

The unimpeded taxi-out time is the nominal or free flow taxi out time, that is, it is the taxi-out time of an aircraft in the absence of any obstacles. The FAA defines the unimpeded taxi-out

time as *the taxi-out time under optimal operating conditions, when neither congestion, weather nor other factors delay the aircraft during its movement from gate to takeoff* (FAA 2002). By this definition, the unimpeded taxi-out time is the average time that an aircraft takes to complete the departure process, if the aircraft spends no time waiting in queues. The service time for each of the steps of the departure process varies due to reasons such as differences in the dispatch stage, routing through different taxiways, differences in taxi speeds and runway assignments, variability in the duration of pushback and engine-start, differences in pilot-controller communications, and differences in staffing in the air traffic control tower facility (Simaiakis 2009). Many of these factors cannot be inferred from the recorded data, and contribute to the variability in unimpeded taxi-out time (Carr et al. 2002).

Federal Aviation Administration Office of Aviation Policy and Plans (2002) uses a linear regression of the observed taxi-out times with the observed departure and arrival queues to determine the unimpeded taxi-out time. Idris et al. (2001) has observed that (1) there is poor correlation of the taxi-out times with arriving traffic, and (2) the taxi-out time of a flight is more strongly correlated with its takeoff queue than the number of departing aircraft on the ground. Carr (2001), Pujet et al. (1999), Simaiakis and Balakrishnan (2009) have also proposed heuristic improvements of the FAA method to better estimate unimpeded taxi-out times.

### 5.1.1. Taxi-out time as a function of the adjusted traffic

The unimpeded taxi time is not directly observed, and needs to be inferred from the data. This section proposes a new method for the estimation of unimpeded taxi-out times from historical archives of taxi-out times at an airport.

Unimpeded taxi-out times can vary with the airline, gate location, aircraft type (smaller aircraft move at faster speeds), destination (international flights tend to have longer taxi procedures and checklists<sup>1</sup> (Simaiakis et al. 2011)), local visibility conditions, runway configuration, and runway assignment (Lee et al. 2010, Simaiakis 2009). All major runway configurations at EWR primarily use a single runway for departures. Therefore, for simplicity, the unimpeded taxi-out time distribution is estimated for each airline at EWR for given visibility conditions and runway configuration, using data from 2011. The proposed estimation method can also be applied to any cluster of similar flights, as has been shown for Philadelphia International (PHL) Airport (Simaiakis 2013).

<sup>1</sup>Finalized data on the aircraft weights and balance, passenger count, cargo loads, and security information are communicated to the aircraft during taxi-out. The updated information is used to calculate the takeoff performance. The taxi and before-takeoff checklists are also completed during taxi-out, along with the takeoff briefing by the Captain (Midkiff et al. 2009).

Idris et al. (2001) and Clewlow et al. (2010) showed that the taxi-out time of an aircraft is poorly correlated to the number of aircraft taxiing-out at the time of its pushback, because inter-aircraft interactions depend on their relative positions, speeds and downstream restrictions. Similarly, other aircraft may push *after* the aircraft and yet get ahead of it in the departure queue. For these reasons, Idris et al. (2001) proposed the concept of a *takeoff queue*, defined as the number of aircraft taking off between the pushback and takeoff times of an aircraft, and developed a probabilistic model to estimate the takeoff queue and taxi-out times for each pushback. Despite the correlation between taxi-out times and takeoff queues, the estimates are biased in practice: Aircraft that taxi slowly (or have a long pre-departure procedure) tend to trail another aircraft in the departure sequence. This selection-bias results in the fastest aircraft having a zero takeoff queue. In other words, the aircraft with zero takeoff queues will not be representative of the average aircraft.

A better indicator of taxi-out time of an aircraft is the *effective traffic* it sees, namely, the sum of the number of aircraft taxiing out when it pushes back, and the number of aircraft that pushback while it is still traveling to the departure runway. Then, the unimpeded taxi-out time is observed as long as there is no increase in taxi-out time with an increase in the effective traffic.

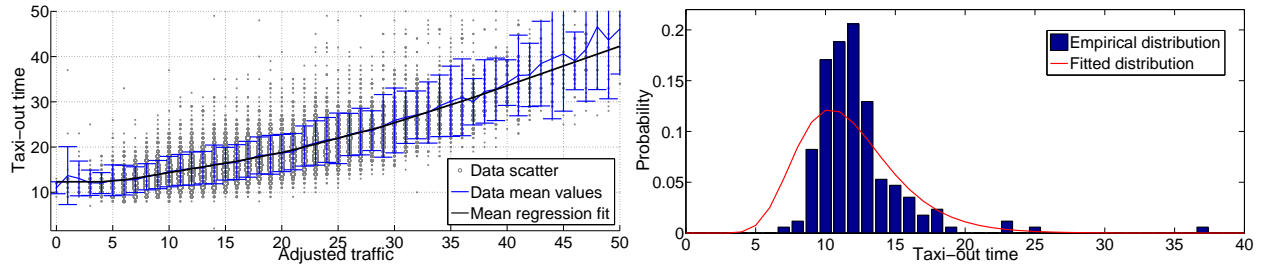
The effective traffic metric takes into account both the aircraft taxiing-out when a flight pushes back, and the traffic that gets added while it is traveling to the runway. Low values of effective traffic imply that the aircraft taxis out unimpeded; any aircraft pushing back after it reaches the departure queue cannot not be sequenced in front of it.

The taxi-out time is expected to be a convex non-decreasing function of the effective traffic. Each aircraft needs a minimum amount of time to reach the departure runway. If an aircraft sees a low value of effective traffic, the probability of being impeded by another aircraft is low; this probability increases as the effective traffic increases. All flights in the effective traffic that pushed back before an aircraft and were not overtaken by it, or that pushed back after it and were faster would be ahead of it in the departure runway sequence. Each additional flight in the effective traffic of an aircraft is likely to be one more aircraft ahead of it in the departure runway sequence, and impose a delay approximately equal to its service time at the runway.

The effective traffic metric assumes a knowledge of when aircraft enter the departure queue, which is not available from the ASPM data. For this reason, the adjusted traffic,  $N_{\text{adj}}(l)$ , for an aircraft  $l$  is defined as the sum of the aircraft taxiing out when it pushes back, and the number of aircraft that pushback while it is taxiing out (Clewlow 2010). For low traffic levels, an aircraft is likely to spend very little time in the departure queue, and the effective and adjusted traffic values are almost the same.

Figure 2 (left) shows the relationship between the adjusted traffic and the taxi-out time for flights of ExpressJet Airlines, a regional partner of Continental/ United Airlines at EWR in 2010. The scatter plot, along with the mean and standard deviation of taxi-out times are shown for all values of the adjusted traffic. The mean taxi-out time appears to be a convex non-decreasing function of the adjusted traffic. This relation was observed in all cases considered; taxi-out time is therefore conjectured to be a convex function of the adjusted traffic.

Figure 2 (left) also shows that the data-scatter for the each value of adjusted traffic has a longer tail for larger taxi-out times. This behavior is to be expected as taxi-out times, even the unimpeded ones, can only be as short as the physics of the process allows, but can grow large, in the event of a slow taxi-out process (for example, because of low speeds, processing taxi and before-takeoff checklists after pushing back, or mechanical issues). This observation is consistent with the those of Carr (2001), Khadilkar (2011), who recommend Erlang or Lognormal distributions for the unimpeded taxi-out time.



**Figure 2** (Left) Empirical data and regression fit of taxi-out times as a function of adjusted traffic, and (Right) Empirical and fitted distribution of the unimpeded taxi-out times, for flights of ExpressJet Airlines in runway configuration 22L | 22R at EWR in 2010.

The next step in the analysis is the fitting of a convex, nondecreasing function that estimates the taxi-out time in terms of the adjusted traffic, to the data in Figure 2 (left). Given  $m$  pairs of measurements  $(N_{\text{adj}}(l), \tau_l)$ , denoted  $(u_1, y_1), \dots, (u_m, y_m)$ , the goal is to determine a convex, non-decreasing function  $f : \mathbb{R} \rightarrow \mathbb{R}$  that estimates the mean  $\tau = f(N_{\text{adj}})$ . Since  $N_{\text{adj}}$  only takes natural number values ( $\mathbb{N}_0$ ), only the values  $f(0), f(1), \dots, f(n)$ , where  $n = \max(N_{\text{adj}})$ , need to be estimated. The function  $f$  is simply a piecewise linear function of  $N_{\text{adj}}$ , and the monotonicity and convexity constraints are imposed at the points  $0, 1, \dots, \max(N_{\text{adj}})$  by comparing the values and the slopes of subsequent pieces.  $f$  is given by the solution to the following convex optimization problem:



$$\begin{aligned}
 & \text{minimize} && \sum_{i=1}^m (\hat{y}_i - y_i)^2 \\
 & \text{subject to} && \hat{y}_i = f(u_i), \quad i = 1, \dots, m \\
 & && f(i+1) \geq f(i), \quad i = 0, \dots, (n-1) \\
 & && f(i+1) - f(i) \geq f(i) - f(i-1), \quad i = 1, \dots, (n-1)
 \end{aligned} \tag{3}$$

The results of the regression fit are shown in Figure 2 (left). It is noted that the estimated function simply smooths out the mean values of the raw data. Figure 2 (left) shows that the fitted function is flat for  $0 \leq N_{adj} \leq 4$ . Therefore, observations for which  $0 \leq N_{adj} \leq 4$  correspond to the unimpeded taxi-out times for ExpressJet Airlines. A log-normal distribution is fit to the empirical data, with mean equaling the estimated value in the flat region ( $f(0)$ ) and standard deviation equaling the standard error of the estimated mean. The resulting distribution and the empirical one are shown in Figure 2 (right). The unimpeded taxi-out times for other major airlines at EWR have also been estimated for other runway configurations using the same method (Simaiakis 2013).

## 5.2. Ramp and taxiway interactions

In prior work (Simaiakis and Balakrishnan 2009), the authors modeled the impact of ramp and taxiway interactions on an aircraft pushing back at time  $t$  as  $\tau_{\text{taxiway}} = \alpha R(t)$ . Combining this model with Equation (2),

$$\tau_{\text{travel}} = \tau_{\text{unimped}} + \alpha R(t) \tag{4}$$

In the above equations, the term  $\alpha R(t)$  is a linear term used to model the interactions among departing aircraft on the ramps and taxiways.  $\alpha$  is a parameter that depends on the airport and the runway configuration, and can be interpreted as the expected number of times an aircraft stops (for example, at a point of congestion) multiplied by the expected duration of each stop. The probability of an aircraft stopping increases with the number of aircraft traveling towards the departure queue at time of its pushback. For instance, consider the expected number of stops being 0.2 multiplied by the number of aircraft traveling. Equation (4) has been previously validated using ASPM data, and successfully applied to different airports (Lee et al. 2010, Nakahara 2012, Simaiakis and Pyrgiotis 2010). Khadilkar (2011) has also validated the linear dependence of the travel time with the level of traffic using high-fidelity surface surveillance data from Boston Logan (BOS) airport. In this paper, parameter  $\alpha$  is chosen such that the predicted median taxi-out time matches the actual median taxi-out time. The reason for this choice is that there is no information on traffic management initiatives which impose significant extraneous delays on aircraft, and the median taxi-out time is less sensitive to these outliers.

### 5.3. Expected runway schedule

As seen in Section 4, the output of Module 1 is the runway schedule. Assuming fixed unimpeded taxi-out times, equal to their expected values, the travel time (from gate to the takeoff queue) of aircraft  $l$  is given by the equation:

$$\mathbb{E}[\tau_l^{\text{travel}}] = \mathbb{E}[\tau_l^{\text{unimped}}] + \alpha R(t_{pb}(l)) \quad (5)$$

where  $\mathbb{E}[\tau_l^{\text{unimped}}]$  is the mean unimpeded taxi-out time of the airline that operates aircraft  $l$  and  $R(t_{pb}(l))$  is the number of traveling aircraft at the time of pushback,  $t_{pb}(l)$ , of aircraft  $l$ .  $R(t)$  is generated adaptively. Combining Equations (1) and (5), the total taxi-out time of aircraft  $l$ ,  $\mathbb{E}[\tau_l]$ , is given by:

$$\mathbb{E}[\tau_l] = \mathbb{E}[\tau_l^{\text{unimped}}] + \alpha R(t_{pb}(l)) + \mathbb{E}[D_l] \quad (6)$$

## 6. Queuing delay estimation (Module 2)

This section discusses the queuing model used for predicting delays at the departure runway(s). External conditions (arrival throughput, downstream restrictions, etc.) dynamically change the characteristics of the departure throughput, but do not explain all of its variability. The departure process is a probabilistic process, impacted by factors, such as controllers' and pilots' decisions, aircraft performance, aborted takeoffs, runway closures, etc.

### 6.1. Runway queuing model

The proposed approach can be used to model either the whole runway system or each individual runway, depending on data availability and the desired model fidelity.

The service rate,  $\mu(t)$ , is defined as the number of departing aircraft that can take off from the runway(s) modeled per 15-minute interval. Four fundamental assumptions are made in the model of the runway service process:

1. Aircraft arrive at the queue according to the runway schedule calculated by Module 1.
2. The service rate is assumed to follow a time-dependent (dynamic) Erlang distribution.
3. There is finite queuing space for the departing aircraft to wait in.
4. Aircraft in the departure queue are served on a First-Come-First-Served (FCFS) basis.

The proposed framework models both the mean and variance of the departure throughput, which have been shown to be important when calculating queuing delays in the context of airport systems (Gupta 2010, Hansen et al. 2009). Inspection shows that this system is a  $D(t)/E_k(t)/1$  one with finite queuing space  $C$ . The finite queuing space  $C$  is assumed here for computational tractability, and not for modeling the limited space available for queuing aircraft near the departure runway(s).

Each portion of a day that a particular runway configuration is in use is divided into time periods of equal duration,  $\Delta$ , and indexed with  $i = 1, 2, \dots, T$ . A throughput distribution of type Erlang, with rate  $k_i\mu_i$  and shape  $k_i$ , is provided for each time period  $i$ , based on the operating conditions. For each time window  $i$ , the set of aircraft arrival times at the departure queue,  $S(i)$ , is known from Module 1. These arrival times are not necessarily uniformly spaced.

Module 2 models a single departure runway in the case of EWR (Runway 4L/22R). If there are multiple runways and the runway assignments are known, each of them can be modeled with a queuing system (Lee et al. 2010). Alternatively, in the absence of runway assignment data, two departure runways have been successfully modeled with a single queuing system with good results, for Philadelphia (PHL) and Charlotte (CLT) airports (Simaiakis 2013).

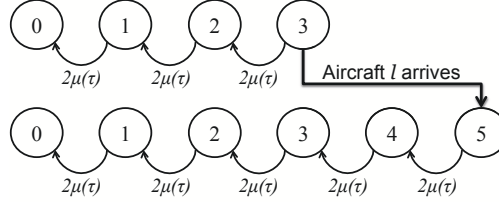
The runway queuing system is modeled as a discrete-time Markov Chain. Service completion of an Erlang process with shape  $k$  and rate  $k\mu$  is represented with  $k$  stages of exponentially distributed random variables with rate  $k\mu$ . Each such stage is called a *stage-of-work*. State  $q$  of the Markov chain denotes that there are  $q$  stages-of-work to be completed at the runway, i.e., there are  $\min(1, q)$  aircraft in service and  $\max(\lceil(q - k)/k\rceil, 0)$  aircraft in the departure queue. The notation used in this section is as follows:

- $l$ : Index of each aircraft.
- $\Delta$ : Duration of each time window, set to 15 minutes.
- $i$ : Index of each time window.
- $(k_i, k_i\mu_i)$ : Shape and rate parameters of Erlang service time distribution in time period  $i$ .
- $C$ : Queuing space of the queuing system, measured in number of aircraft.
- $Q_l$ : Stages-of-work to be completed immediately after  $l^{\text{th}}$  aircraft's arrival to empty the system.
- $p_{Q_l}(j)$ : Probability that  $Q_l$  takes the value  $j$ .
- $c_j$ : Inter-arrival time at the runway between the  $(j - 1)^{\text{th}}$  and  $j^{\text{th}}$  aircraft.
- $C_l = \sum_{j=1}^l c_j$ : Time of arrival of the  $l^{\text{th}}$  aircraft.
- $S_l$ : Total time that aircraft  $l$  spends in the queuing system, including time-in-service.
- $D_l$ : Queuing delay that aircraft  $l$  experiences.
- $Z_0$ : Number of aircraft in queue at the beginning of the first time window.
- $f_\nu(x), F_\nu(x)$ : P.D.F., C.D.F of r.v.  $X$  drawn from Poisson distribution with parameter  $\nu$ .

### 6.1.1. Static service time distribution

The runway service process is assumed to be static and described by a single Erlang distribution with parameters  $(k, k\mu)$ . The system is observed at the epochs of arrival,  $C_l$ , of each aircraft  $l$ . Figure 3 shows an example of the state of the Markov chain that an aircraft encounters, assuming

that there are exactly 3 stages-of-work to be completed before the arrival of  $l^{\text{th}}$  aircraft in the system, and an Erlang distribution with shape 2 for the departure throughput. Upon the arrival of  $l^{\text{th}}$  aircraft, 2 stages-of-work are added and the system (instantaneously) transitions to state 5.



**Figure 3** Markov chain transition at the time of arrival of the  $l^{\text{th}}$  aircraft.

The probabilities  $p_{Q_l}(j)$  are calculated next.  $p_{ij}^l$  is defined as the probability that there are  $i$  stages-of-work to be completed immediately after the arrival of the  $l^{\text{th}}$  aircraft, given that there were  $j$  stages-of-work to be completed immediately after the arrival of the  $(l-1)^{\text{th}}$  aircraft. Then, since the system evolves as a Poisson process in time  $c_l$ :

$$p_{ij}^l = \Pr(Q_l = i | Q_{l-1} = j) = \begin{cases} 1 - \sum_{z=0}^{j-1} \frac{e^{-k\mu c_l} \cdot (k\mu c_l)^z}{z!} & \text{if } i = k \\ \frac{e^{-k\mu c_l} \cdot (k\mu c_l)^{j+k-i}}{(j+k-i)!} & \text{if } i = k+1, \dots, j+k, i < kC \\ \sum_{z=0}^{j-i+k} \frac{e^{-k\mu c_l} \cdot (k\mu c_l)^z}{(z)!} & \text{if } i = kC, i \leq j+k \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

In Equation (7), an *event* is defined as the completion of a stage-of-work in the Markov chain of the system. The time between each pair of consecutive events has an exponential distribution with parameter  $(k\mu)$ . The inter-arrival times between events are i.i.d. Suppose  $z$  denotes the number of events during time  $c_l$ ; then  $z = j + k - i$ . Up to  $j$  stages-of-work can be completed in time  $c_l$ , so  $0 \leq z \leq j$ . Since the arrival of the  $l^{\text{th}}$  aircraft adds  $k$  stages-of-work,  $i \geq k$ . The probability of  $z < j$  events (that is,  $i > k$ ) is therefore given by the Poisson distribution with parameter  $(k\mu c_l)$ . The constraint that  $z \leq j$  implies that the probability of emptying the queuing system ( $z = j$  events) is equal to one minus the probability of having  $0, 1, \dots, j-1$  events. Finally, if  $j \geq kC - k$ , sufficient space is created for the  $l^{\text{th}}$  aircraft when it arrives. Thus, for  $k(C-1) \leq j \leq kC$ , the system transitions to state  $i$  with the sum of the probability of  $0, 1, \dots, j - k(C-1)k$  events. This is in contrast to standard queuing system models, where the new customer is rejected from the system if there is no queuing space. In the present situation, selecting a very low value of  $C$  can “boost” the throughput of the system by forcing stage-of-work completions at each epoch. The choice of  $C$  is a tradeoff between computational tractability and accuracy. Numerical experiments suggest that for  $C = 100$ , the behavior is close to that of infinite queuing space.

Using the Poisson distribution notation, Equation (7) can be written as follows, where  $\nu = k\mu c_l$ :

$$P(\nu) = \begin{matrix} & 0 & 1 & \dots & kC-k & kC-k+1 & \dots & kC-1 & kC \\ \begin{matrix} 0 \\ \vdots \\ k-1 \\ k \\ k+1 \\ \vdots \\ kC-1 \\ kC \end{matrix} & \left( \begin{matrix} 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 1 & 1-F_\nu(0) & \dots & 1-F_\nu(kC-k-1) & 1-F_\nu(kC-k) & \dots & 1-F_\nu(kC-2) & 1-F_\nu(kC-1) \\ 0 & f_\nu(0) & \dots & f_\nu(kC-k-1) & f_\nu(kC-k) & \dots & f_\nu(kC-2) & f_\nu(kC-1) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & f_\nu(1) & f_\nu(2) & \dots & f_\nu(k) & f_\nu(k+1) \\ 0 & 0 & \dots & F_\nu(0) & F_\nu(1) & \dots & F_\nu(k-1) & F_\nu(k) \end{matrix} \right) \end{matrix} \quad (8)$$

$P(\nu)$  is a stochastic matrix, as its columns sum up to 1 for all  $k$  and  $\nu$ . Equation (7) implies:

$$p_{Q_l}(i) = \sum_{j=i-k}^{j=kC} p_{ij}^l p_{Q_{l-1}}(j) \text{ for } i = k, k+1, \dots, kC \quad (9)$$

$$\Rightarrow \mathbf{p}_{Q_l} = P(\nu) \mathbf{p}_{Q_{l-1}}, \text{ where } \mathbf{p}_{Q_l} = [p_{Q_l}(0), p_{Q_l}(1), \dots, p_{Q_l}(kC)]'. \quad (10)$$

The initial condition is the queue encountered by the first aircraft arrival,  $Z_0$ . If  $Z_0 = 0$ , then the first aircraft will bring the system to state  $k$  upon its arrival. A non-empty initial condition may correspond to runway configuration changes, when aircraft are lined up at the new departure runway prior to its use. The state of the queue when the first aircraft arrives is:

$$p_{Q_0}(i) = \begin{cases} 1 & \text{if } i = k \cdot Z_0 \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

### 6.1.2. Dynamic service time distribution

Equation (10) is still valid for dynamic service time distributions by using the appropriate parameters  $(k_i, k_i \mu_i)$  for the time window  $i$  in which inter-arrival time  $c_l$  falls. For  $\nu_i = k_i \mu_i c_l$ ,

$$\mathbf{p}_{Q_l} = P(\nu_i) \mathbf{p}_{Q_{l-1}} \quad (12)$$

In contrast to the static case, the state space of the queuing system can change in the dynamic case. For example, given two runway throughput distributions, with shapes  $k_1$  and  $k_2$ , where  $k_1 \neq k_2$ , the state-space of the Markov chain corresponding to the first distribution is  $\{0, 1, \dots, k_1 C\}$  and the second is  $\{0, 1, \dots, k_2 C\}$ . Transitioning from the first throughput distribution to the second will require mapping the probabilities of states  $\{0, 1, \dots, k_1 \cdot C\}$  to those of  $\{0, 1, \dots, k_2 \cdot C\}$ .

## 6.2. Queuing delay

### 6.2.1. Static service process distribution

Given the probabilistic state of the Markov chain described by  $\mathbf{p}_{Q_l}$  at the time of arrival, the moments of the queuing time of each aircraft can be calculated assuming a static service process, that is, an Erlang distribution with parameters  $(k, k\mu)$ . The first two moments are given by:

$$\mathbb{E}[S_l] = \mathbb{E}[\mathbb{E}[S_l | Q_l]] = \mathbb{E}\left[\frac{i}{k\mu} | Q_l = i\right] = \sum_{i=0}^{kC} \frac{i \cdot p_{Q_l}(i)}{k\mu} \quad (13)$$

$$\mathbb{E}[S_l^2] = \mathbb{E}[\mathbb{E}[S_l^2|Q_l]] = \mathbb{E}\left[\frac{i(i+1)}{\mu^2} \mid Q_l = i\right] = \sum_{i=k}^{kC} \frac{i(i+1)}{(k\mu)^2} \cdot p_{Q_l}(i) \quad (14)$$

The mean and variance of the queuing delay can be calculated similarly by subtracting the service time for the  $l^{\text{th}}$  aircraft, that is,  $k$  stages-of-work, and then calculating the moments.

### 6.2.2. Dynamic service process distribution

When the service process is dynamic, the parameters  $(k, k\mu)$  can change across time windows. The expected delay of a given state cannot be calculated because the parameters of the associated transitions change dynamically. Ball et al. (2007), Gupta (2010), Pyrgiotis (2011) have previously noted this problem with dynamic models. This paper builds on the approach from Gupta (2010) for D(t)/M(t)/1 systems. For each aircraft, the *effective queue*,  $\tilde{q}_l(j)$ , is an estimate of the *physical queue* that aircraft  $l$  faces. It measures the queue in terms of aircraft and not stages-of-work, and does not depend on future service time distributions.

$$\tilde{q}_l(j) = \sum_{j=k_i}^{k_i C} \frac{(j - k_i) \cdot p_{Q_l}(j)}{k_i} \quad (15)$$

The effective queue,  $\tilde{q}_l(j)$ , is calculated for each aircraft at the moment of its arrival at the departure queue. Subsequently, the equivalent deterministic service process is used to calculate the total time that it will take for the effective queue,  $\tilde{q}_l(j)$ , to be dissipated. This time is known as the *effective delay*,  $\tilde{d}_l(j)$ . In this way, each aircraft in the effective queue of aircraft  $l$  is served with the service rate that applies at the time of their takeoff. The variance of queuing delays are calculated similar to the static case, but with parameters  $(k, k\mu)$  corresponding to the those at the time of arrival of the aircraft at the departure queue.

### 6.3. Estimation of the departure capacity distributions

Figure 4 shows the dependence of the departure throughput on the departure demand for (VMC; 22L | 22R), the most frequently used runway configuration at EWR in 2011. The average throughput is seen to stabilize once there are around 17 aircraft taxiing out, and stay at around 10 aircraft/15 min until there are more than 25 aircraft on the ground. Plots such as these are known as *saturation plots*, and have been previously used by Shumsky (1997) and Pujet et al. (1999). For measuring the impact of other explanatory variables, periods of sustained departure demand are extracted and a regression tree analysis is conducted (Hand et al. 2001, Simaiakis 2013). Figure 5 shows the regression tree for the departure throughput ( $T$ ) as a function of the departure demand ( $N$ ), the arrival throughput ( $A$ ), and the surrogate RAPT value ( $S_{RAPT}$ ). The regression tree indicates that EWR is in saturation for  $16 \leq N \leq 31$ .

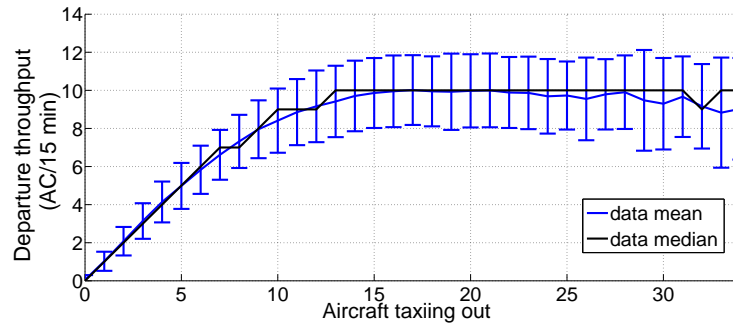


Figure 4 EWR saturation plot for configuration (VMC; 22L | 22R) in year 2011.

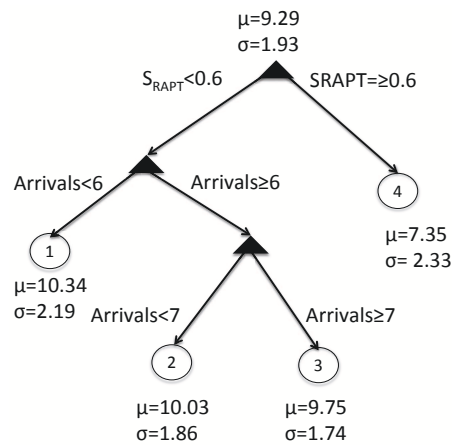


Figure 5 Regression tree with departure throughput parametrized by route availability and arrival throughput at EWR in the (VMC; 22L | 22R) configuration.

Data from time-periods during which the airport is saturated are used to estimate the departure throughput as a function of the arrival throughput and the route availability (RAPT). Regression trees are used to obtain information on higher order moments of the throughput distribution. The resulting (pruned) tree is shown in Figure 5. For example, for arrival throughput less than 6 aircraft/15 min and  $S_{RAPT}$  value lower than 0.6, the departure throughput estimates are between 10 aircraft/15 min and 10.5 aircraft/15 min. Given the variance of the throughput measurements in this region, these estimates are merged into a single leaf with expected throughput ( $\mu$ ) of 10.34 aircraft/15 min and standard deviation ( $\sigma$ ) of 2.19 aircraft/15 min.

Regression trees such as the one in Figure 5 are used to characterize the dynamics and stochasticity of the departure process. The route availability and the arrival throughput at each 15-minute period map to a leaf of the tree. Each leaf of the tree is associated with an empirical distribution, denoted  $f_{rw}$ , that corresponds to all the observations satisfying its conditions. Assuming that the service times are generated from an Erlang distribution with parameters  $(k, k\mu)$ , these parameters can be estimated using an approximation based on the method of moments. The output is the

**Table 1** Standard deviation of the distributions  $f_{rw}$  and  $f_{rm}$ .

Distribution No.	1	2	3	4
$\mathbf{f}_{rw}$	2.19	1.86	1.74	2.33
$\mathbf{f}_{rm}$	2.30	1.87	1.84	1.95

Poisson distribution satisfied by the  $k^{\text{th}}$  arrival of the exponential distribution with rate  $(k\mu)$  that matches the first moment and has the smallest absolute error of the second moment of  $f_{rw}$  (Simaiakis 2013). The modeled distributions, denoted  $f_{rm}$ , are found to be similar to the four empirical distributions, as can be seen in the comparison of their standard deviations in Table 1.

#### 6.4. Module 2 outputs

Module 2 takes a runway schedule as input from Module 1, and outputs the queuing delay for each flight and the departure throughput (Sections 6.2 and 6.3). For each 15-minute time period,  $i$ , given the modeled parameters of the service time distribution  $(k_i, k\mu_i)$  and the inter-arrival times  $c_i$ , it calculates the queue state probability vector  $\mathbf{p}_{Q_i}$ . For each aircraft  $l$ , it calculates its effective queue length  $\tilde{q}_l(j)$  and its effective queuing delay, denoted  $\tilde{d}_l(j)$ . The takeoff time of each aircraft is given by  $C_l + \tilde{d}_l(j)$ , that is, the sum of its arrival time at the departure queue and its effective queuing delay. Repeating this process for each aircraft yields the expected takeoff schedule. From the runway schedule and the expected takeoff schedule, airport performance characteristics such as the expected throughput, expected queue length and the expected number of aircraft on the ground can be calculated.

It is worth noting that Module 2 is independent from Module 1. Therefore, given the runway schedules of two departure runways, Equation (12) can be solved independently for each one to calculate their performance characteristics. This approach has been applied successfully to Detroit airport (DTW) in Lee et al. (2010).

#### 6.5. Sensitivity to modeling assumptions

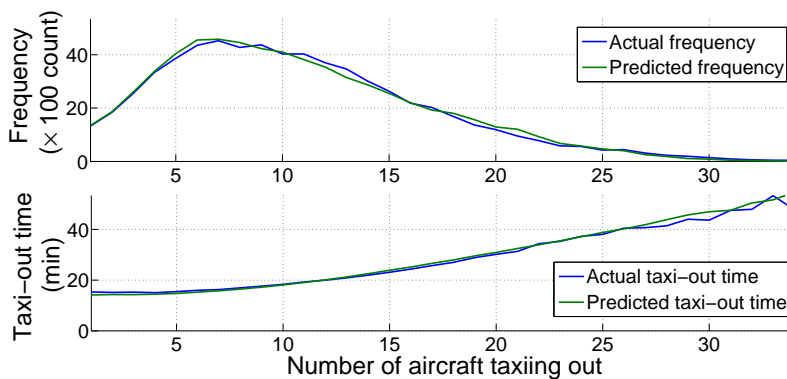
The two main assumptions made in the development of the model above were: (1) The stochasticity of the travel times can be ignored, and the travel times and the runway schedule can be assumed to be deterministic (Equation 5); and that (2) the service times can be assumed to follow an Erlang distribution. In particular, while the second assumption has been widely used in airport literature (Hengsbach and Odoni 1975, Hansen et al. 2009, Kivestu 1976, Malone 1995), it has not, to the best of our knowledge, been validated using actual data. Simaiakis (2013) checks this assumption using surface surveillance data for BOS, and shows that the Erlang distribution, while convenient from an analysis standpoint, may not be the best fit to the data. However, Simaiakis (2013) also shows using Monte Carlo simulations that the predictions of the queuing model being proposed in this paper are not sensitive to either of the assumptions mentioned above.



## 7. Model results for EWR

This section presents the model results for the most frequently used runway configuration at EWR in 2011, (VMC; 22L | 22R). The unimpeded taxi-out times were estimated using 2011 data, as explained in Section 5.1. The throughput distributions were also estimated from 2011 data, as explained in Section 6.3. As noted in Section 5.2,  $\alpha = 0.27$  min/aircraft was calculated so that the predicted median taxi-out time equaled the actual median time (18 min). The input to the model was the pushback schedule in 2011.

Figure 6 shows the frequencies with which different congestion states were observed in the operational data and predicted by the model, for 2011. Figure 6 also shows the expected taxi-out times as a function of the number of aircraft taxiing-out at the time of pushback, for both the actual operations and the model predictions. Table 2 contains more detailed statistics on the number of aircraft and the taxi times in different congestion levels. Table 2 and Figure 6 both show that the model accurately predicts both the frequency of different congestion states, and the taxi-out times at each state. In particular, the number of flights predicted to be at congestion states greater than or equal to 15 is only 2% more than the actual value, and their predicted mean taxi-out time is over-estimated by only 2%.



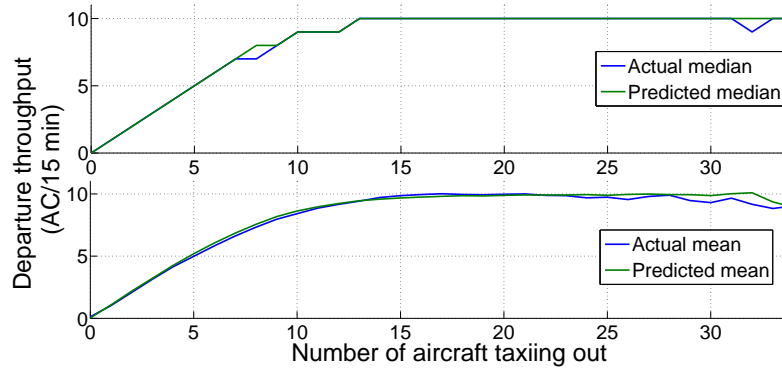
**Figure 6** (Top) Actual and predicted frequency of each state  $N$ , and (bottom) actual and predicted average taxi-out times as a function of the state  $N$  at the time of pushback, for runway configuration 22L | 22R at EWR in 2011.

Figure 7 shows the predicted throughput of configuration 22L | 22R at EWR in 2011 as a function of the congestion state  $N$ . The model accurately predicts both the mean and median throughputs in all traffic conditions. The model does not explicitly predict the variance of the departure throughput, although it is accounted for in the underlying service time distributions.

Table 3 evaluates the predictions of the congestion state ( $N$ ) and the 15-minute throughput for each minute during which there was traffic on the ground in the actual data ( $N(t) > 0$ ). It shows

**Table 2** Aggregate taxi time predictions for EWR runway configuration 22L | 22R in 2011.

Congestion level	Number of flights		Mean taxi-out time	
	Actual	Model	Actual (min)	Model (min)
All	65,990	65,977	20.41	20.19
( $N \leq 8$ )	27,387	27,964	15.89	15.10
( $9 < N \leq 14$ )	22,594	21,683	19.51	19.47
( $N \geq 15$ )	16,009	16,330	29.42	29.84

**Figure 7** Actual and modeled throughput of all states  $N$  for EWR runway configuration 22L | 22R in year 2011: Mean (top); Median (bottom).

the mean error (ME), the mean absolute error (MAE) and the root mean square error (RMSE) of the predictions. The negative mean error of -0.2 units indicates that the congestion state is under-predicted by 0.2 (aircraft taxiing out) on average. The condition  $N(t) > 0$  removes data-points with no traffic. Table 3 also presents results for times when  $N(t) > 10$ , because these are more relevant to the problem of congestion. It is seen that when  $N(t) > 10$ , the congestion state is under-predicted by 0.9 units. The under-prediction is due to the fact that very high congestion states are also related to rare events such as safety incidents, temporary runway closures, mechanical problems or constraints at destination airports, which are not modeled.

Predictions of the 15-minute throughput are also evaluated in Table 3. The errors are lower than the errors of the congestion state predictions because the average throughput is not very sensitive to the exact value of  $N(t)$ , especially when the throughput is saturated. At each minute  $t$  during 2011, the model predicts both the number of aircraft on the ground and the throughput with a mean absolute error of 1.7 aircraft and 1.1 aircraft/15 min respectively.

### 7.1. Comparison to a deterministic model

The results of the proposed model can be compared to an alternative one with deterministic service times. In this setting, each aircraft is served deterministically with the average service time of the time-window  $i$  in which it takes off. The runway schedule used in the previous section is used in this analysis, and the results are compared.

**Table 3** Statistics on predictions of the congestion level and throughput for the 22L | 22R configuration at EWR in 2011.

	N(t) > 0			N(t) ≥ 10		
	ME	MAE	RMSE	ME	MAE	RMSE
Congestion state (AC)	-0.20	1.71	3.03	-0.91	3.00	4.5
Throughput (AC/15 min)	0.00	1.14	1.60	-0.32	1.37	1.85

Table 4 presents the results for the taxi-out time predictions using the deterministic model. The deterministic model underestimates the average taxi-out time, overestimates the number of aircraft that takeoff in light congestion ( $N \leq 8$ ), and (analogously) underestimates the number of aircraft that takeoff in higher congestion states ( $N \geq 15$ ) by 12%. The deterministic model transitions to higher congestion states less often than the stochastic one, resulting in fewer flights facing long queues. As a result, it underestimates the predicted average taxi-out time by 1.5 minutes. However, the predicted taxi-out time given high congestion is only 0.5% less than the actual value. This similarity at high traffic levels is to be expected, as deterministic and stochastic models converge to the same delays for flights that face a long queue (Koopman 1972).

**Table 4** Aggregate taxi-out time predictions using a deterministic model for the 22L | 22R configuration at EWR in 2011.

Congestion level	Number of flights		Mean taxi-out time	
	Actual	Model	Actual (min)	Model (min)
All	65,990	65,977	20.41	19.10
( $N \leq 8$ )	27,387	30,277	15.89	14.77
( $9 < N \leq 14$ )	22,594	21,494	19.51	18.47
( $N \geq 15$ )	16,009	14,204	29.42	29.27

Figure 8 shows the taxi-out time of each aircraft as a function of its takeoff queue, that is, the number of aircraft that takeoff between its pushback and takeoff time. For low values of the takeoff queue, ( $N_{\text{takeoff}} \leq 6$ ) both the deterministic and stochastic models overestimate the taxi-out time compared to the actual data. As explained in Section 5.1.1, the overestimation is due to selection-bias in the data.

For intermediate values of the takeoff queue ( $8 < N_{\text{takeoff}} < 18$ ), the stochastic model predictions are quite accurate, while the deterministic model consistently underestimates the taxi-out time. As the values of the takeoff queue increase, the predicted taxi-out time of both models converge, as shown in Figure 8. Simaiakis (2013) has a more detailed discussion of the reasons for the inferior performance of the deterministic model.

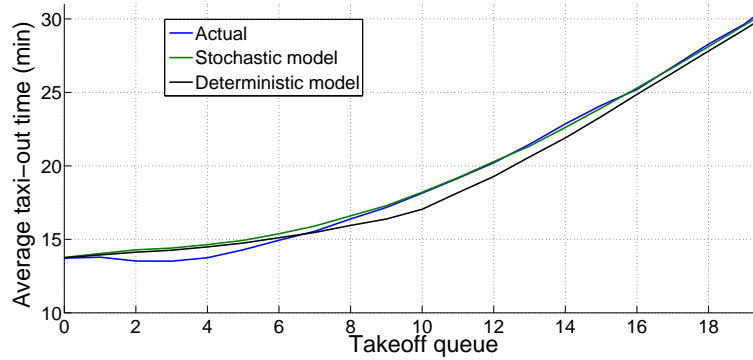


Figure 8 Actual and predicted average taxi-out times as a function of the takeoff queue of each aircraft.

## 8. Predictive ability of the proposed model

This section discusses the ability of the model to predict operations for years other than 2011, which was used to identify the model parameters. In particular, the predictive performance for 2007 and 2010 are evaluated. The inputs were the actual (realized) pushback schedules, arrival demand and *RAPT* values during the times that runway configuration 22L | 22R was operational. Since there was no *RAPT* data available for the year 2007, the predictions for 2007 are expected to be worse than those for 2010.

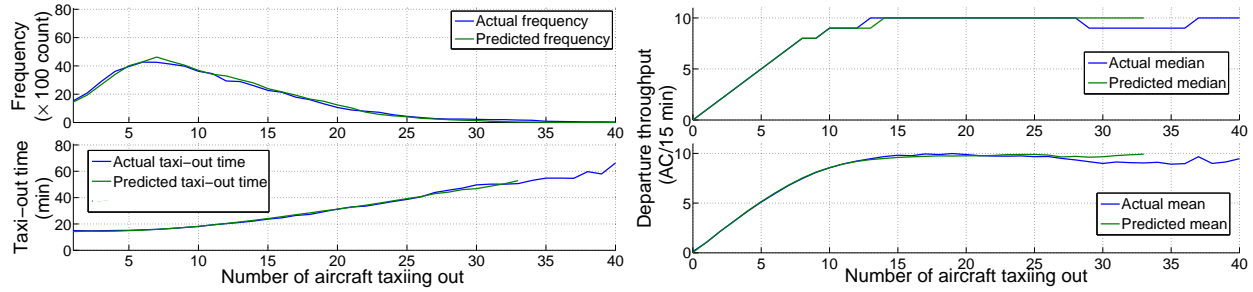
### 8.1. Predictions for EWR in 2010

In this section, the model is used to predict operations in the 22L | 22R configuration in 2010. Figure 9 (left) shows the actual and predicted frequencies of each congestion state, and the average taxi-out times as a function of the number of aircraft taxiing out at the time of pushback. Qualitatively, the 2010 predictions are seen to match actual data as well as they did for 2011. Figure 9 (right) shows the predicted and actual mean and median throughputs in 2010 as a function of the number of departing aircraft on the ground. The model is seen to overpredict both the mean and the median throughput in highly congested states, and to underestimate the taxi-out times. The underlying reason is that the throughput in 2010 is seen to decrease for congestion states higher than 25, which is not predicted by the model.

Table 5 shows the predicted average taxi-out times at different states. The number of flights in higher congestion states ( $N \geq 15$ ) as well as their delays are underestimated.

### 8.2. Predictions of taxi-out times for individual days

Given the pushback times of flights, the proposed model estimates their travel time to the runway, the amount of time that they spend in the departure queue, the overall state of the airport surface (for example, the number of departures on the ground), and the length of the departure queue.



**Figure 9** Validation of model predictions for 22L | 22R configuration at EWR in year 2010. (Left) [Top] Actual and modeled frequencies of all states,  $N$ ; [Bottom] Actual and modeled average taxi-out times as a function of the state  $N$  at the time of pushback. (Right) [Top] Actual and modeled mean throughput; [Bottom] Actual and modeled median throughput.

**Table 5** Aggregate taxi time predictions for EWR runway configuration 22L | 22R in 2010.

Congestion level	Number of flights		Mean taxi-out time	
	Actual	Model	Actual (min)	Model (min)
All	63,633	63,585	20.83	20.30
( $N \leq 8$ )	27,945	27,942	15.46	15.19
( $9 < N \leq 14$ )	19,530	20,234	19.49	19.69
( $N \geq 15$ )	16,158	15,409	31.73	30.32

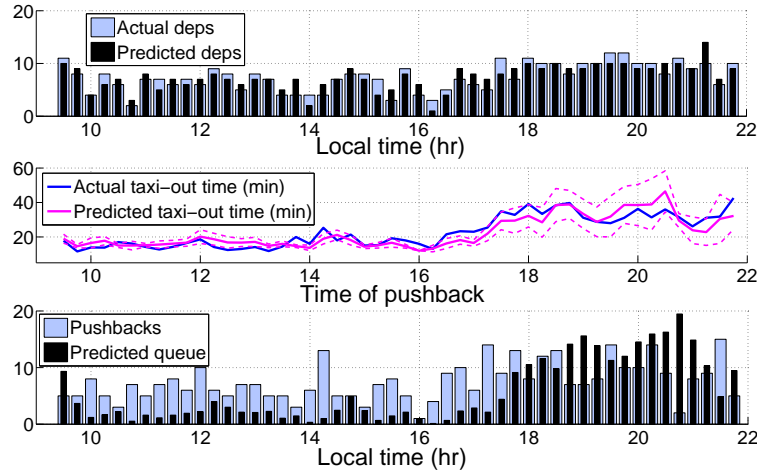
**Table 6** Statistics on predictions of the congestion level and throughput for the 22L | 22R configuration at EWR in 2010.

	$N(t) > 0$			$N(t) \geq 10$		
	ME	MAE	RMSE	ME	MAE	RMSE
Congestion state (AC)	-0.33	1.78	3.49	-1.55	3.45	5.56
Throughput (AC/15 min)	-0.01	1.12	1.61	-0.34	1.41	1.92

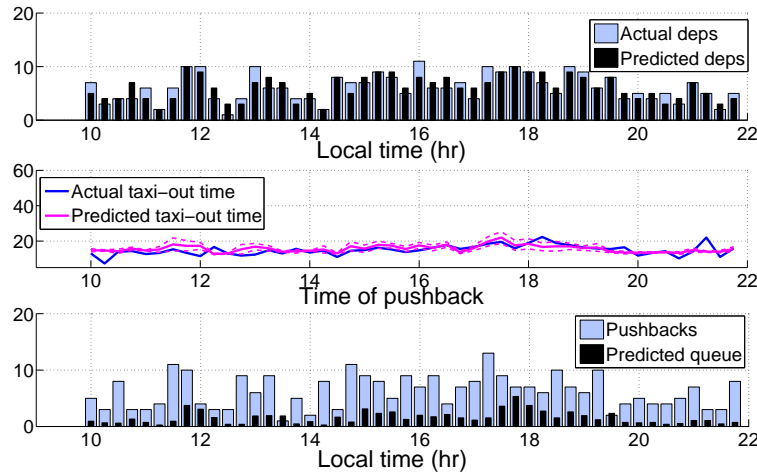
While the aggregate performance of the model in 2010 has been addressed, this section evaluates its ability to predict operations on a particular day when 22L | 22R was in use.

Figures 10, 11 and 12 show the predictions for a 13-hour period on three days in EWR, along with the observed data. The top plot on each figure shows the observed and predicted number of departures in a 15-minute window, the middle plots show the average taxi-out times of the flights that push back in the corresponding 15-minute window, and the bottom plots show the average predicted departure queue size and number of pushbacks for each 15-minute window in that day. The estimated variance of the queuing delays is used to provide a confidence interval for the average taxi-out time predictions. The dashed lines represent the standard deviation of the queuing delay for a flight that is expected to takeoff in the middle of each 15-minute interval.

Figure 10 (bottom) shows a predicted departure queue of at least 10 aircraft between 1745 and 2115 hours. The persistently long queue induces an increased variance of the queuing delays, as can be seen from the middle plot. For flights that push back between 1945 and 2045 hours, the



**Figure 10** Predictions of departure throughput, average taxi-out times and departure queue lengths in each 15-min interval over a 13-hour period on Thursday, August 5, 2010.

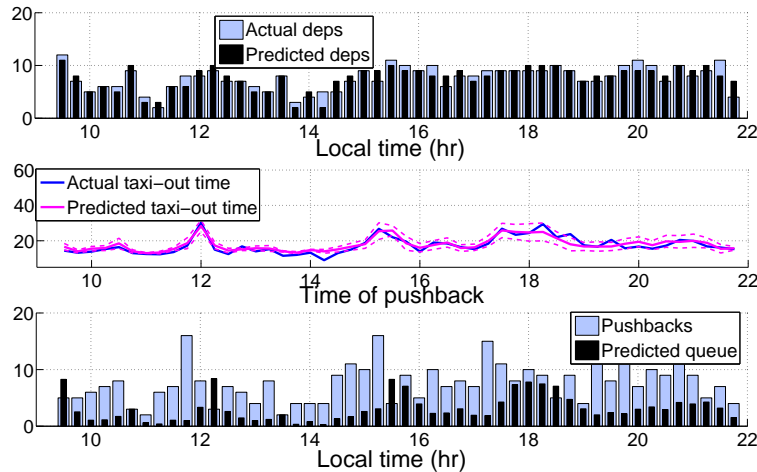


**Figure 11** Predictions of departure throughput, average taxi-out times and departure queue lengths in each 15-min interval over a 12-hour period on Friday, 26 November, 2010.

predicted standard deviation of the queuing delay is around 25 minutes. The average taxi-out time is under-predicted between 1600 and 1800 hours, and overestimated between 1930 and 2030 hours. It is difficult to accurately predict taxi-out times without any updates in such a dynamic and stochastic system, when pressure on the queue is sustained for a long time.

The throughput is accurately predicted, and during peak times (1800 to 2100 hours), the error is at most 2 aircraft/15 min. However, each error propagates to the taxi-out times of all later flights, since the queue never empties. The standard deviation of the queuing delays, albeit approximate, provides useful insights on the delay risks of such a dense pushback schedule.

Figures 11 and 12 offer similar insights. Figure 11 shows a day with much less demand, which experiences lower average delays and variability than the one in Figure 10. Figure 12 shows a



**Figure 12** Predictions of departure throughput, average taxi-out times and departure queue lengths in each 15-min interval over a 13-hour period on Wednesday, 8 December, 2010.

day with high, but not uniformly distributed, demand. At 1200 hours, the predicted and actual taxi-out times are both very high (30 minutes). However, the departure push is very short, and the queue does not build up, keeping the queuing delay variance small. By contrast, for the time period between 1730 and 1815 hours, shorter but more uncertain expected delays are predicted.

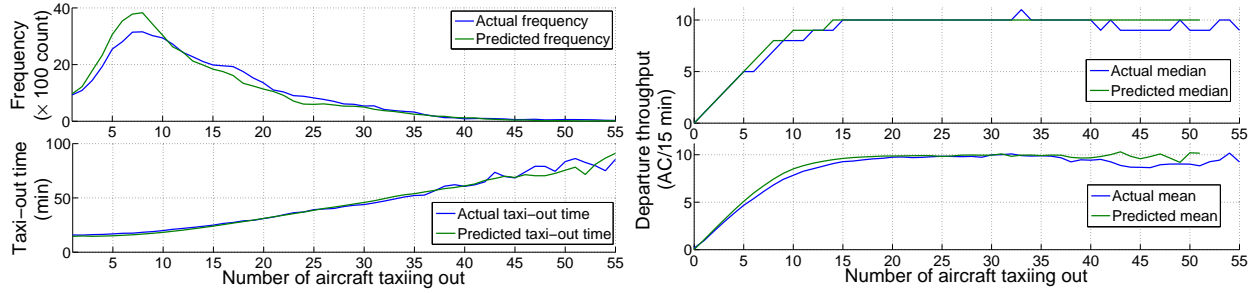
### 8.3. Predictions for EWR in 2007

The proposed model (calibrated with 2011 data) is also used to predict congestion and delays in 2007, the worst recent year in terms of delays. Figure 13 (left) shows the actual and predicted frequencies of each congestion state, and the average taxi-out times as a function of the number of aircraft taxiing out at the time of pushback. The quality of the predictions is seen to deteriorate compared to 2010 and 2011, due to the absence of route availability data. However, the model still correctly predicts a much higher average taxi-out time than in 2010 or 2011. In particular, Table 7 shows that for the most congested states, the taxi-out time predictions are quite accurate, and the model underestimates the number of flights by only 15%. Thus, the model provides useful information for the magnitude and severity of delays expected as a result of the very high demand in 2007.

Figure 13 (right) shows the predicted mean and median throughput of segment (VMC; 22L | 22R) at EWR in 2007 as a function of the number of departures on the ground. The model over-predicts the throughput in intermediate congestion states, and as a result, delays are underestimated.

#### 8.3.1. Variability of queuing delays

The predictions of the model can be used for evaluating the delay uncertainty of a very busy schedule, such as the one in EWR in 2007. Figure 14 shows the scatter plot of the expected queuing



**Figure 13** Validation of model predictions for 22L | 22R configuration at EWR in year 2007. (Left) [Top] Actual and modeled frequencies of all states,  $N$ ; [Bottom] Actual and modeled average taxi-out times as a function of the state  $N$  at the time of pushback. (Right) [Top] Actual and modeled mean throughput; [Bottom] Actual and modeled median throughput.

**Table 7** Aggregate taxi time predictions for EWR runway configuration 22L | 22R in 2007.

Congestion level	Number of flights		Mean taxi-out time	
	Actual	Model	Actual (min)	Model (min)
All	55,506	55,704	25.83	23.48
$(N \leq 8)$	17,899	21,424	17.09	15.49
$(9 < N \leq 14)$	15,506	15,640	19.49	19.61
$(N \geq 15)$	22,101	18,640	36.12	35.91

**Table 8** Statistics on predictions of the congestion level and throughput for the 22L | 22R configuration at EWR in 2007.

	$N(t) > 0$			$N(t) \geq 10$		
	ME	MAE	RMSE	ME	MAE	RMSE
<b>Congestion state (AC)</b>	-1.27	2.94	5.84	-2.78	5.09	8.36
<b>Throughput (AC/15 min)</b>	0.01	1.32	1.89	-0.27	1.57	2.17

delay (in min) and predicted variance (in  $\text{min}^2$ ) for each flight in the 22L | 22R configuration, in 2007 and 2011. The figure shows the impact of the much higher pushback demand in 2007 on the variability of queuing delays.

#### 8.4. Taxi-out time predictions for individual flights

In addition to aggregate comparisons, it is interesting to see how the model predicts individual taxi times. The predicted and observed taxi-out times for the flights in the 22L | 22R configuration are compared for 2007, 2010 and 2011. Figure 15 shows the cumulative distribution of the prediction error  $E_l$  for each flight  $l$ , defined as

$$E_l = \tau_l^{\text{pred}} - \tau_l^{\text{obs}} \quad (16)$$

Figure 15 shows that 2010 and 2011 have similar error distributions, and that the taxi-out time predictions are accurate to within  $\pm 10$  min for 88% of the flights. The errors are much higher in



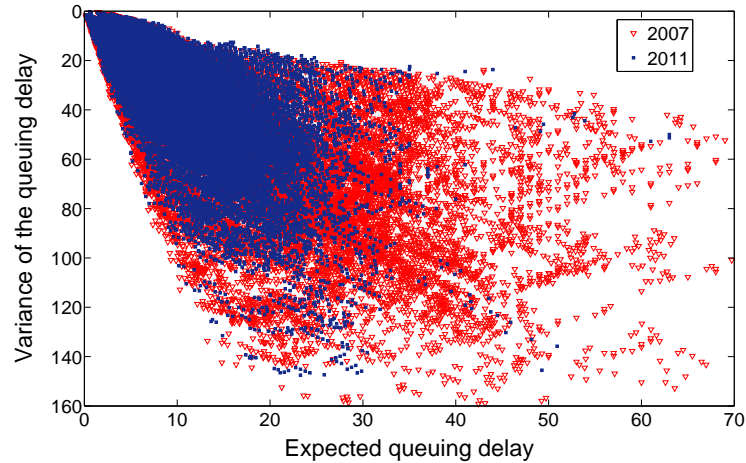
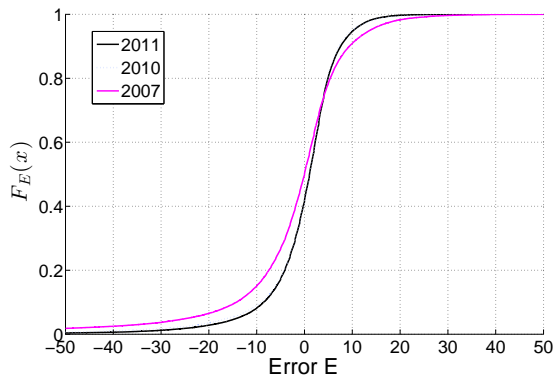


Figure 14 Predicted impact of high demand on the variability of the queuing delays.



Year	ME (min)	MAE (min)	RMSE (min)
2011	-0.22	5.51	8.81
2010	-0.53	5.82	9.88
2007	-2.35	8.53	16.09

Figure 15 (Left) Taxi-out time prediction error distributions; (Right) Statistics on individual taxi-out time predictions, for individual flights in 22L | 22R at EWR.

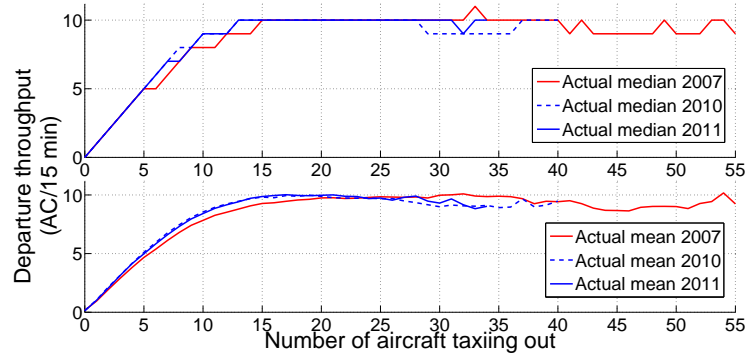
2007, and the taxi-out time predictions are within  $\pm 10$  min of the actual values for only 73% of the flights. The taxi-out time is under-predicted by 50 minutes or more for approximately 2% of the flights (approximately 1,000) in 2007. There were significantly fewer such flights in 2010 and 2011 (approximately 300), which may be a consequence of the 3-hour tarmac delay rule, that imposes large penalties on airlines for taxi-out times of more than 180 min.

The table in Figure 15 lists the Mean Error (ME), Mean Absolute Error (MAE) and the Root Mean Square Error (RMSE) for these predictions. It shows that the taxi-out time predictions for individual flights were reasonably good in 2010 and 2011. The next section investigates some of the possible reasons for the poor predictions in 2007.

### 8.5. Airport performance in 2007

Figure 16 shows the mean and median throughput observed in the 22L | 22R configuration at EWR in 2007, 2010 and 2011. The throughput of the airport is clearly seen to change from 2007 to 2011,

especially at intermediate values of the number of aircraft taxiing out. Thus, although the average capacity of the airport remains 10 aircraft/15 min across the three years, the expected throughput as a function of the number of aircraft taxiing out changes. Route availability, data about which is unavailable for 2007, is hypothesized to be a driver of this different behavior. Other possible reasons for the difference across the three years include differences in procedures, frequency of downstream constraints and traffic management initiatives, or regulations (such as the tarmac delay rule).



**Figure 16** Actual throughput in all states  $N$  for EWR runway configuration 22L | 22R in years 2007, 2010 and 2011: (Top) Mean; (Bottom) Median.

### 8.6. Predictions for the 4R | 4L runway configuration

The methods used and the resulting insights are similar to those discussed earlier in this paper for the 22L | 22R configuration. The model parameters are estimated using 2011 data, and the model is used to predict operations in 2007 and 2010. Table 9 summarizes the main results of the taxi-out time predictions for individual flights. The average taxi-out time predictions for 4R | 4L are found to be better than those for 22L | 22R in all three years. The better performance is likely because 4R | 4L tends to not be used during convective weather, and thus the predictions are less impacted by the lack of route availability information in 2007. The taxi-out time prediction errors for individual flights in 2010 (the test dataset) are also found to be smaller than in 2011 (the training dataset).

**Table 9** Prediction statistics for individual taxi-out times for the 4R | 4L configuration at EWR in 2007, 2010 and 2011.

Year	Number of flights	Average taxi-out time (min)	ME	AME	RMSE
2011	37,132	22.73	-0.50	5.58	8.67
2010	39,785	22.86	0.17	5.48	8.17
2007	34,378	29.55	-0.12	7.63	11.27

Table 10 summarizes the actual and predicted mean and median taxi-out times for the 22L | 22R and 4R | 4L configurations for 2007, 2010 and 2011. The model is found to predict airport performance reasonably well for both major runway configurations across different years.

**Table 10** Summarized prediction results for the 22L | 22R and 4R | 4L configurations at EWR in 2007, 2010 and 2011.

Congestion level	Year	Mean taxi-out time		Median taxi-out time	
		Actual (min)	Model (min)	Actual (min)	Model (min)
22L   22R	2011	20.41	20.19	18	18
	2010	20.83	20.30	18	18
	2007	25.83	23.48	21	19
4R   4L	2011	22.73	22.23	20	20
	2010	22.86	23.03	20	21
	2007	29.55	29.43	25	24

## 9. Extensions

The proposed modeling approach has been applied to the study of different airport layouts and runway configurations, at Charlotte (CLT), La Guardia (LGA), Boston (BOS) and Philadelphia (PHL) airports (Simaiakis 2013). It has also been used in the development of surface congestion control algorithms for BOS (Simaiakis et al. 2013). The models were used to predict congestion levels and average taxi-out times. In all the above instances, the departure runways were modeled as a single server. However, the prediction errors of individual taxi-out times were found to be significantly larger for CLT, where two independent departure runways are used. Runway assignments are not available in the ASPM data, but can be detected through surface surveillance systems such as ASDE-X (Federal Aviation Administration 2010). Using such information, the proposed approach can be used to model each runway with an individual server, as has been shown for Detroit (DTW) airport (Lee et al. 2010).

## 10. Conclusions

This paper proposed and validated a new analytical queuing model of the departure processes at airports that can be used for strategic planning and tactical predictions. The model consists of a stochastic and dynamic queuing model of the departure runway(s), based on the transient analysis of  $D(t)/E_k(t)/1$  queuing systems. A new method for estimating the unimpeded taxi-out time distributions was proposed, and techniques to estimate both the distribution of inter-departure times and the throughput distribution were proposed. The model parameters were calibrated using 2011 data, and shown to predict taxi-out times, departure throughput and congestion levels accurately

for the two major runway configurations at EWR in the years 2007, 2010 and 2011. In addition to the average delays, the model was shown to predict the variance of the predicted queuing delays. The proposed modeling approach has been used with similar success to develop models of other major airports, and for the development of congestion control strategies.

## Acknowledgments

This work was supported in part by NSF CPS:Large:ActionWebs, award number 0931843.

## References

- Ball, M., C. Barnhart, G. Nemhauser, A. Odoni, C. Barnhart, G. Laporte. 2007. Air transportation: Irregular operations and control. *Handbooks in Operations Research and Management Science* **14**(1) 1–67.
- Carr, F., A. Evans, J.P. Clarke, E. Feron. 2002. Modeling and control of airport queueing dynamics under severe flow restrictions. *Proceedings of the American Control Conference*.
- Carr, F. R. 2004. Robust Decision-Support Tools for Airport Surface Traffic. Ph.D. thesis, Massachusetts Institute of Technology.
- Carr, F.R. 2001. Stochastic modeling and control of airport surface traffic. Master's thesis, Massachusetts Institute of Technology.
- Clarke, J.P., T. Melconian, E. Bly, F. Rabbani. 2007. MEANS MIT Extensible Air Network Simulation. *Simulation* **83**(5) 385.
- Clewlow, R. 2010. A Prediction Model for Aircraft Surface Delays. Tech. rep., MIT.
- Clewlow, R.R., I. Simaiakis, H. Balakrishnan. 2010. Impact of Arrivals on Departure Taxi Operations at Airports. *AIAA Guidance, Navigation and Control Conference and Exhibit*.
- DeLaura, R., S. Allan. 2003. Route selection decision support in convective weather: a case study of the effects of weather and operational assumptions on departure throughput. *5th Eurocontrol/FAA ATM R&D Seminar, Budapest, 23–27 June 2003*.
- FAA. 2002. Federal Aviation Administration, Office of Aviation Policy and Plans. Documentation for the Aviation System Performance Metrics (ASPM).
- FAA. Accessed February 2012. Aviation System Performance Metrics database. <http://aspm.faa.gov/aspm/ASPMframe.asp>.
- Federal Aviation Administration. 2010. Fact Sheet of Airport Surface Detection Equipment, Model X (ASDE-X).
- Federal Aviation Administration Office of Aviation Policy and Plans. 2002. Documentation for the Aviation System Performance Metrics: Actual versus Scheduled Metrics. Tech. rep.
- Feron, E. R., R. J. Hansman, A. R. Odoni, R. B. Cots, B. Delcaire, W. D. Hall, H. R. Idris, A. Muharremoglu, N. Pujet. 1997. The Departure Planner: A conceptual discussion. Tech. rep., Massachusetts Institute of Technology.

- Gupta, S. 2010. Transient analysis of D(t)/M(t)/1 queuing system with applications to computing airport delays. Master's thesis, Massachusetts Institute of Technology.
- Hand, D. J., H. Mannila, P. Smyth. 2001. *Principles Of Data Mining*. The MIT Press.
- Hansen, M., T. Nikoleris, D. Lovell, K. Vlachou, A. Odoni. 2009. Use of Queuing Models to Estimate Delay Savings from 4D Trajectory Precision. *Eighth USA/Europe Air Traffic Management Research and Development Seminar*.
- Hengsbach, G., A.R. Odoni. 1975. Time dependent estimates of delays and delay costs at major airports. Tech. rep., Cambridge, Mass.: MIT, Dept. of Aeronautics & Astronautics, Flight Transportation Laboratory.
- Idris, H., J. P. Clarke, R. Bhuvu, L. Kang. 2001. Queuing Model for Taxi-Out Time Estimation. *Air Traffic Control Quarterly*.
- Idris, H. R. 2001. Observation and analysis of departure operations at Boston Logan International Airport. Ph.D. thesis, Massachusetts Institute of Technology.
- Idris, H. R., B. Delcaire, I. Anagnostakis, W. D. Hall, N. Pujet, E. Feron, R. J. Hansman, J. P. Clarke, A. R. Odoni. 1998. Identification of flow constraint and control points in departure operations at airport systems. *AIAA Guidance, Navigation and Control Conference*.
- Jacquillat, A., A. R. Odoni. 2012. A Study of Airport Congestion at JFK and EWR. *5th International Conference on Research in Air Transportation*.
- Khadilkar, H.D. 2011. Analysis and modeling of airport surface operations. Master's thesis, Massachusetts Institute of Technology.
- Kivestu, P. A. 1976. Alternative methods of investigating the time dependent M/G/k queue. Master's thesis, Massachusetts Institute of Technology.
- Koopman, B.O. 1972. Air-terminal queues under time-dependent conditions. *Operations Research* **20**(6) 1089–1114.
- Lee, H., I. Simaiakis, H. Balakrishnan. 2010. A Comparison of Aircraft Trajectory-Based and Aggregate Queue-Based Control of Airport Taxi Processes. *Digital Avionics Systems Conference*. IEEE.
- Long, D., D. Lee, J. Johnson, E. Gaier, P. Kostiuk. 1999. Modeling Air Traffic Management Technologies with a Queuing Network Model of the National Airspace System. Tech. rep., NASA Langley Research Center, Hampton, VA. Technical Report NASA/CR-1999-208988.
- Malone, K. M. 1995. Dynamic queuing systems : behavior and approximations for individual queues and for networks. Ph.D. thesis, Massachusetts Institute of Technology.
- Midkiff, A. H., R. J. Hansman, T. G. Reynolds. 2009. Airline flight operations. P. Belobaba, A. Odoni, C. Barnhart, eds., *The Global Airline Industry*. John Wiley and Sons, United Kingdom.

- Nakahara, A. 2012. Estimating current and future benefits of airport surface congestion management techniques. Master's thesis, Massachusetts Institute of Technology.
- Odoni, A.R., J. Bowman, D. Delahaye, J.J. Deyst, E. Feron, R.J. Hansman, K. Khan, J.K. Kuchar, N. Pujet, R.W. Simpson. 1997. Existing and required modeling capabilities for evaluating ATM systems and concepts. Tech. rep.
- Pujet, N., B. Delcaire, E. Feron. 1999. Input-output modeling and control of the departure process of congested airports. *AIAA Guidance, Navigation, and Control Conference and Exhibit, Portland, OR* 1835–1852.
- Pyrgiotis, N. 2011. A Stochastic and Dynamic Model of Delay Propagation Within an Airport Network For Policy Analysis. Ph.D. thesis, Massachusetts Institute of Technology.
- Pyrgiotis, N., K.M. Malone, A. Odoni. 2011. Modelling delay propagation within an airport network. *Transportation Research Part C: Emerging Technologies* .
- Shumsky, R. A. 1995. Dynamic statistical models for the prediction of aircraft take-off times. Ph.D. thesis, Massachusetts Institute of Technology.
- Shumsky, R.A. 1997. Real-Time Forecasts of Aircraft Departure Queues. *Air Traffic Control Quarterly* **5**(4).
- Simaiakis, I. 2009. Modeling and Control of Airport Departure Processes for Emissions Reduction. Master's thesis, Massachusetts Institute of Technology.
- Simaiakis, I. 2013. Analysis, Modeling and Control of the Airport Departure Process. Ph.D. thesis, Massachusetts Institute of Technology.
- Simaiakis, I., H. Balakrishnan. 2009. Queuing Models of Airport Departure Processes for Emissions Reduction. *AIAA Guidance, Navigation and Control Conference and Exhibit*.
- Simaiakis, I., H. Balakrishnan. 2010. Analysis and Control of Airport Departure Processes to Mitigate Congestion Impacts. *Transportation Research Record: Journal of the Transportation Research Board* 22–30.
- Simaiakis, I., H. Balakrishnan, H. Khadilkar, T.G. Reynolds, R.J. Hansman, B. Reilly, S. Urllass. 2011. Demonstration of Reduced Airport Congestion Through Pushback Rate Control. *9th Eurocontrol/FAA ATM R&D Seminar*.
- Simaiakis, I., H. Balakrishnan, M. Sandberg. 2013. Dynamic control of airport departures: Algorithm development and field evaluation. *IEEE Transactions on Intelligent Transportation Systems* .
- Simaiakis, I., N. Pyrgiotis. 2010. An Analytical Queuing Model of Airport Departure Processes for Taxi Out Time Prediction. *AIAA Aviation Technology, Integration and Operations (ATIO) Conference*.
- Wieland, F. 1997. The detailed policy assessment tool (DPAT). *Proceedings of the Spring INFORMS Conference*.
- Willemain, T. R., H. Fan, H. Ma. 2004. Statistical analysis of intervals between projected airport arrivals. Rensselaer Polytechnic Institute DSES Technical Report No. 38-04-510.