Measurement, Uncertainty, and Data Analysis
Student Worksheet: Number Sense

Answer the following questions, estimating where appropriate. Indicate your answers with uncertainty where possible.

1. If three people are sharing a pie, how much of the pie is the fair share of each person? How much of the pie is each person likely to get, assuming you cut as carefully as you can?

2. The speed of light in a vacuum is defined as exactly 299,792,458 m/s. If this speed is about 0.03% slower in air, what is the speed of light in air?

3. Because of a large mirrors installed by Apollo 11, 14, and 15, we are able to measure the distance to the moon to the nearest millimeter. The distance varies slightly from day to day due to the Moon's elliptical orbit, but the distance is, on average, 384,403 km. If the moon moves 3.5 cm further away from the Earth every year, what will be the distance to the Moon in 100 years?

4. If there are 5 freezers with ice cream sandwiches and 382 students interested in getting some dessert, how many ice cream sandwiches should be in each freezer?

5. Around the time of an election, one pollster reports that Ms. Smith is likely to get 50 % of the vote and Mr. Jones is likely to receive 46% of the vote. The margin of error (SEM) is reported to be ±5%. Is Ms. Smith really ahead? Explain.

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Teacher Notes: Number Sense

The purpose of this activity is to reinforce the common sense approach to uncertainty and error that was taught in the Power Point "Measurement and Error." Students should be beginning to get a sense for the difference between a scientific measurement and a mathematical expression.

1. The "fair share" is a mathematical idea; it can be expressed as exactly −, or . This has no uncertainty, since it does not derive from a measurement of the physical world. However, when we consider the share that each person is likely to receive, we need to think realistically about the irregularities in any pie, the sharpness of any knife, the precision of any measuring device. Answers will vary somewhat, but depending on the care with which the pie is cut, I would report the share any one person is likely to receive as 33 ± 5 %, or maybe even 33 ± 10 %.

2. Even though the speed of light is exactly defined and therefore completely precise, when it is added to a value described as "about" 0.03% of itself, much of the precision is lost. We have $299792458 + 9 \times 10^4$ m/s = 299880000 ± 10000 m/s.

3. Even though the distance to the moon is known at any given instant to within a millimeter, its average value is only reported (and known) to within 1 km. So, we can think of the measurement as 384,403 ± 1 km. When we consider the effect of tides slowing down the Earth's orbit and causing the moon to recede from the Earth, the change is only 3.5 ± 0.1 cm / year x 100 years = 350 ± 10 cm = 3.5 ± 0.1 m = 0.0035 ± 0.0001 km. We have (384,403 ± 1) + (0.0035 ± 0.0001) km = 384,403 ± 1 km. The uncertainty in the distance to the moon is so much greater than the change in the distance that the reported distance does not change.

4. It doesn't make sense to think about a fraction of an ice cream sandwich, so, in order to make sure no one goes without dessert, each freezer should have 77 ice cream sandwiches. If some can have different numbers, you could have 3 freezers with 76 sandwiches and 2 with 77 sandwiches.

5. The two candidates are within the margin of error. While Ms. Smith is ahead in the polls by an amount nearly equal to the margin of error, there is some doubt as to the results.