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From possible individuals to scalar segments

Roger Schwarzschild[1]

> the meaning of a sentence must be
> more like an arrow than a point.
>
> L. Wittgenstein quoted in Portner (1992)

1 Introduction

A comparative serves to locate two or more individuals on a scale associated with a particular gradable predicate. On most analyses, this involves quantification over abstract entities from an ordered realm. Theories differ on the nature of those entities and on their relation to the meanings of gradable predicates. My plan is to explore the consequences of adopting the following pair of theses:

(a) Degree constructions make use of quantification over scalar segments, parts of a scale.

(b) Gradable predicates denote relations between possible individuals. Degrees and segments are introduced with the functional vocabulary. They are defined in terms of possible individuals.

In §2, I introduce ingredients for a semantics based on segments with an analysis of simple phrasal comparatives[2]. The analysis is modeled on neo-Davidsonian event semantics, with segments existentially quantified in place of events. In §3, I show how degrees can be constructed from relations among individuals. The two theses jointly necessitate the presence in comparatives of operators that combine with individual-

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[2] A phrasal comparative is simple if the object of *than* corresponds to the subject of the gradable predicate. This includes *A is bigger than B, You are closer to Washington than me,* and it excludes *You are closer to Washington than Chicago, You threw the ball higher than me, You bought a bigger house than me.*

relational predicates and introduce segments. This necessity sets the stage for a brief discussion in §4 of a typology in which the functional lexicon is the locus of variation. In §5, the two theses come together in a structure for clausal comparatives in English. At bottom is the comparative marker, *–er*, that combines with gradable predicates and introduces segments. The null *wh*-operator traditionally associated with comparatives is a predicate of segments, a segmental modifier (Izvorski 1995). It is raised, producing an expression denoting a set of sets of segments. *than* encodes the greater-than relation and combines two clauses both constructed with *–er* and the null operator (Alrenga, Kennedy, and Merchant 2012). By abstracting over modifiers denoting sets of segments, we account for the quantifier facts that motivated analyses based on degree intervals and degree pluralities (see Beck 2014, Dotlačil and Nouwen 2016, Fleisher 2016, this volume, and references therein). By allowing *–er* some scopal leeway, we gain a mechanism for capturing the facts about modals in comparative clauses for which the π operator was introduced (Heim 2006). The proposal will also allow us to address an empirical puzzle to do with ellipsis in differential comparatives ('10º' in (1) below is called a differential). Bresnan (1973) proposed that a comparative *than* clause, such as in (1) below, is generated as a sister to the comparative marker *–er*. If the *than* clause were generated where it is pronounced, that would require, given assumptions Bresnan made about ellipsis, that a copy of the differential be generated inside the comparative clause, a situation she deemed "semantically incorrect" (Bresnan 1973:388ff). In the meantime, Bhatt and Pancheva (2004) have adduced ample evidence that in fact *than* clauses must be generated where they are pronounced. But the judgment of semantic incorrectness persists and is taken to support the idea that DegP (differential+*–er*) is moved producing an antecedent for ellipsis, as in (2).

(1) It is 10º colder today than it was yesterday
(2) [It is t_1 cold today][10º–*er*]$_1$ [than OP$_1$ it was ⟨t_1 cold⟩ yesterday]

On the proposal to be advanced in §5, differentials that appear in the main clause can have corresponding instantiations in the comparative clause. This is due in part to the decision to quantify over segments, which can be described by differentials. As a result, a differential does not need to move to produce an antecedent for ellipsis. This becomes important when the differential is in the scope of an attitude verb with a de-dicto interpretation as in (3) below:

(3) {Jack and Jill are train enthusiasts. They've been discussing a high-speed freight train planned for their region. They wonder whether the boxcars will be 60ft long, like on the Santa Fe line, or 50ft long, like on the Caroliner. As far as the engine is concerned, Jack and Jill disagree. Jack's expectation is that the engine will be 2 boxcars long. Jill expects it to be one boxcar long}

Jack expects the engine to be one boxcar longer than Jill does.

The differential *one boxcar* needs to be interpreted in the scope of *expect*: there is no actual length of a boxcar at issue here, only expected lengths. If *boxcar* is indeed in the scope of *expect*, it must lie between *expect* and *long* at LF and since both *expect* and *long* are elided in the comparative clause, I conclude that *boxcar* was elided as well. This implies the presence of a differential in the comparative clause.

The thesis in (a) above is closely related to proposals in Faller (2000) and Winter (2005). They analyze adjectives and comparatives in a semantics based on vectors, which have length and direction, like segments. An explanation of which adjectives can be modified by a measure phrase (e.g. *4ft tall*) and which cannot (e.g **4ft short*) is given in those works in terms of the contours of the vector space that the adjective denotes. The result interestingly extends the work begun in Zwarts (1997) on modification of locative prepositions (*2ft behind the desk, *2ft near the desk*). Relatedly, my initial motivation for developing a segmental semantics was the crosslinguistic recruitment of spatial vocabulary for the expression of comparatives and other degree constructions. Some of these facts are mentioned in §2 below where I review the segmental semantics developed in Schwarzschild (2012,2013) and Thomas (2018). In that framework, components of a comparative meaning combine intersectively and so they can be rearranged and recombined. Thomas (2018) capitalizes on this feature in a distributive-morphology grammar in which morphemes which occur separated out in some languages are realized together in other languages, giving rise to crosslinguistically recurring polysemies in which additivity, continuity and comparison are colexified.

The thesis in (b) above also has antecedents. Hoeksema (1983:424) and Bale (2006, this volume) construct degrees outside the meaning of the gradable predicate via higher functional morphemes. When degrees are not introduced as primitives, but are built up in some way, the result is a theory in which one can ask questions about the nature of degrees, such as what kinds of degrees there are and how they get ordered. In §3, I present a new way to derive degrees and I use it to address those questions as well as questions about measurement and degrees that I've always found intractable. What does it mean to say *two pounds* names a degree? Can degrees be added and if so, what does the addition represent? If we take degrees of weight and height to be things named by *two pounds* and *two meters*, what do we say about degrees associated with adjectives for which there aren't numerical measures or, worse, for which there could not be a meaningful numerical measure system? What does it mean to say that, when used as a differential, *five feet* measures the distance between two degrees? And how does that relate to its use as an adjectival modifier (*five feet tall*)?

Abstract entities – cardinalities, degrees, possible worlds, times – all come to life in the functional lexicon. In Klein (1994), this association is formalized by treating verb phrases as mere event predicates and introducing times via aspectual heads. Beck and von Stechow (2015) extend this logic to possible worlds. Similar to the *Asp* operator which locates an event temporally, they assume a *Modl* head which locates an event in a world. The thesis in (b) above extends this logic to degrees but with an important difference. Beck & von Stechow (2015) assume a primitive ontology with times and worlds and use functional morphemes to enter them into the computation. Here and in the abovementioned papers, the functional morphemes 'construct' degrees out of primitives, worlds and individuals.[3]

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[3] In fact, Wellwood (2015) fits the Kleinian model quite well. In that theory, adjectives are predicates of states, a degree domain is presupposed and functional morphemes relate

2 Scale segments

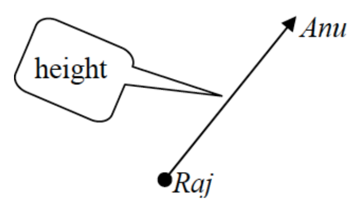
A scale segment is a triple consisting of a measure function that assigns degrees and two degrees in the range of the function. The triple consisting of Anu's height, Raj's height and the height function (HT) is a scalar segment. The first element of the triple represents the **start** of the segment and the second element of the triple is the **end** of the segment. A segment is said to be **rising** if its end is higher than its start. To say that Anu is taller than Raj, is to say that the segment that starts with Raj's height and ends with Anu's height is rising. Using the notation defined in (4) below, we can formalize (5) as in (6):

(4) Segmental semantic notation

σ	variable over scalar segments
START(σ)	the first element of σ (a degree)
END(σ)	the second element of σ (a degree)
μ_σ	the third element of σ (a measure function)
$\nearrow(\sigma)$	σ is a rising segment

(5) Anu is taller than Raj.

(6) $\exists\sigma$ [START(σ) = μ_σ(Raj) $\wedge$ END(σ) = μ_σ(Anu) $\wedge$ μ_σ = HT $\wedge$ $\nearrow(\sigma)$]



A semantics based on scalar segments allows for flexibility in how meanings are parceled out. The comparative marker, *–er*, in (5) above plausibly encodes the rise predicate, '↗' of (6), while START is encoded in *than.* In many languages, these two

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the adjectival states to degrees. Yet another theoretical possibility is for adjectives to have degree-based meanings but not degree arguments. Svenonius & Kennedy (2006) hypothesize that "degree arguments are … introduced by functional morphology in the extended projection of the adjective" by which they mean that an adjective of type $\langle e,d\rangle$ combines with a functional head to produce an expression of type $\langle d,\langle e,t\rangle\rangle$. Cresswell (1976), Klein (1991:679) and Rullmann (1995:125), on the other hand, show how degrees can be constructed out of individuals and worlds, and Anderson & Morzycki (2015) make the case for understanding degrees to be sets of possible states. In those papers, the construction is not associated with a step in the compositional semantics.

meanings are jointly encoded. Compare the following Q'eqchi' examples[4] from Kockelman (2018) formed around the adjective *aal* 'heavy':

(7)

syen	*liibr*	*aal*	*li*	*winq*
100	pounds	heavy	the	man

'The man weighs 100 pounds.'

(8)

wib'	*liibr*	*aal*	*li*	*winq*	[chi–r–u	li	ixq]
2	pounds	heavy	the	man	PREP–E3S–RN	the	woman

'The man is two pounds heavier than the woman.'

The only indication that (8) is a comparative is the presence of the prepositional phrase *chiru li ixq*, in which the starting point of the comparative is introduced, so *chiru* plausibly encodes '↗' and START.

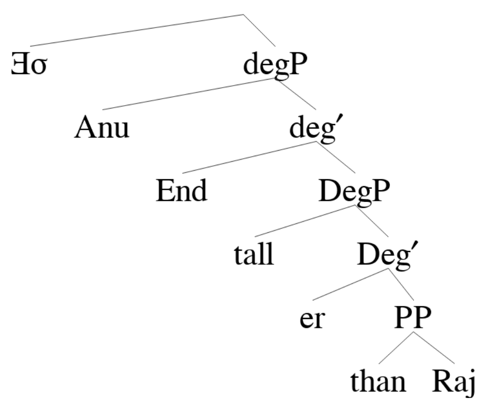
One can discern a similarity between (5)/(6) and event semantic formalizations like in (9)-(10), with segments playing the role of events.

(9) Anu called Raj

(10) $\exists e$ [Agent(e) = Anu $\wedge$ Call(e) $\wedge$ Patient(e) = Raj]

Pursuing this analogy, we adopt the structure and meanings below[5] in which we conceive of 'End' and *than* as akin to thematic role predicates:

(11)



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[4] PREP = preposition, E = ergative case marker, RN = relational noun.

[5] The structure is based on the one in Thomas (2018:61-62). The AP *tall* is merged in the specifier of DegP, following Lechner (2004). Thomas' chief interest lies in a particular pattern of syncretism across languages. Segmental semantics allows him to give meanings to the elements of an abstract syntactic structure. Heads in the structure undergo the restructuring operations of Distributed Morphology before being realized by morphophonological forms. I've simplified by replacing those heads with the morphemes that realize them.

(12) $[\![than]\!]^{g,c} = \lambda x\, \lambda \sigma\, .\ \text{START}(\sigma) = \mu_\sigma(x)$

(13) $[\![-er]\!]^{g,c} = \lambda \sigma\, .\nearrow(\sigma)$

(14) $[\![tall]\!]^{g,c} = \lambda \sigma\, .\mu_\sigma = \text{HT}$

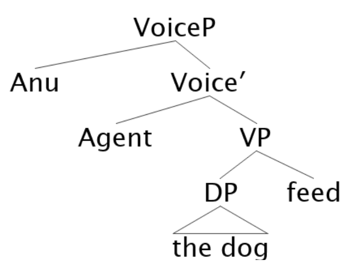
(15) $[\![\text{End}]\!]^{g,c} = \lambda x\, \lambda \sigma\, .\ \text{END}(\sigma) = \mu_\sigma(x)$

(16) $[\![\exists\sigma]\!]^{g,c} = \lambda P_{\langle\sigma,t\rangle}\ \exists\sigma\ P(\sigma)$

Using σ for the type of segments, we observe that the PP, the comparative marker *–er* and the adjective *tall* are all of type $\langle\sigma,t\rangle$ and can compose intersectively. Giving us:

(17) $[\![[_{\text{DegP}}\ \text{taller than Raj}]]\!]^{g,c} = \lambda \sigma\, .\ \mu_\sigma = \text{HT} \wedge \nearrow(\sigma) \wedge \text{START}(\sigma) = \mu_\sigma(\text{Raj})$

Anu is the individual whose degree marks the endpoint of the segment. The phrase *Anu* is introduced in the Specifier of degP, just as the agent argument is introduced in the Specifier of *v*P in analyses of argument structure influenced by Kratzer's (1996) analysis of external arguments. To combine a VP meaning of type $\langle\varepsilon,t\rangle$ (function from events to truth-values) with the type $\langle e,\langle\varepsilon,t\rangle\rangle$ meaning of the agent thematic voice head at the Voice′ node in the structure below, Kratzer introduces a rule she calls 'Event Identification'.



Following Thomas (2018), we'll employ a corresponding rule of 'Segment Identification' to combine 'End' with DegP at the deg′ node. The rule is stated as follows:

(18) Segment Identification
Let α be a node with two daughters, β and γ. Let β be of type $\langle e, \langle\sigma, t\rangle\rangle$ and γ of type $\langle\sigma,t\rangle$, then $[\![\alpha]\!]^{g,c} = \lambda x\, \lambda \sigma\, .\ [\![\beta]\!]^{g,c}(x)(\sigma) \wedge [\![\gamma]\!]^{g,c}(\sigma)$

With (18) we now have:

(19) $[\![[_{\text{Deg}'}\ \text{End taller than Raj}]]\!]^{g,c} =$

$$\lambda x\, \lambda \sigma\, .\ \text{END}(\sigma) = \mu_\sigma(x) \wedge \mu_\sigma = \text{HT} \wedge \nearrow(\sigma) \wedge \text{START}(\sigma) = \mu_\sigma(\text{Raj})$$

From there, by function argument application, we get the logical equivalent of (6):

(20) $\exists\sigma\, [\text{END}(\sigma) = \mu_\sigma(\text{Anu}) \wedge \mu_\sigma = \text{HT} \wedge \nearrow(\sigma) \wedge \text{START}(\sigma) = \mu_\sigma(\text{Raj})]$

In languages like Q'eqchi' in which the comparative is marked by the presence of an adpositional phrase introducing the starting point of the comparison, the adposition has a meaning like in (21) which includes ↗ and START:

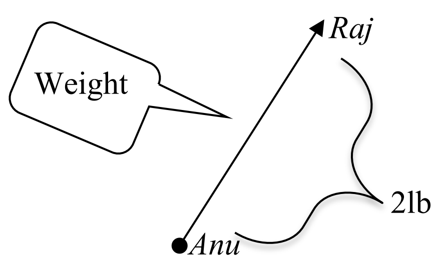
(21) $\lambda x\ \lambda\sigma\ .\ \nearrow(\sigma) \wedge \textsc{start}(\sigma) = \mu_\sigma(x)$

The intersective architecture of event semantics was designed to account for the logic of modifiers, syntactic adjuncts to the verb that can be added or dropped without affecting grammaticality (Davidson 1967). Like Q'eqchi', Navajo has a postposition, −*lááh*, used in PPs marking the starting point of a comparison. −*lááh* introduces the starting point (START), but it also marks the clause as a comparative of superiority (↗). Replacing -*lááh* 'beyond' with =*gi* 'at' leads to an equative meaning and replacement with -*'oh* produces a less-than comparative (↘). Bogal-Allbritten (2013:§3.3) establishes that these PPs can, and in many cases must, combine as adjuncts, occurring at some distance from the adjectival verbs they modify[6].

Navajo standard marking -*lááh* has a spatial use with a meaning glossed as 'beyond'. Across a variety of languages, the starting point of a comparison is indicated with an adposition or a case marker, often one that can be used for a Source thematic role. I borrowed the phrase "the starting point of a comparison" from a discussion of Greenlandic languages in which the starting point is marked with ablative case (de Mey 1976). The proposal in (11) began as an attempt to make sense of these origins.[7]

In the Q'eqchi' comparative in (8) above, the measure phrase *wib' liibr* '2 lbs' describes the *difference* in weight between the man and the woman. This leads to formalization along the following lines:

(22) Raj is 2 lbs heavier than Anu.
(23) $\exists\sigma\ [\textsc{end}(\sigma) = \mu_\sigma(\text{Raj}) \wedge \mu_\sigma = \textsc{wt} \wedge \nearrow(\sigma) \wedge \textsc{start}(\sigma) = \mu_\sigma(\text{Anu}) \wedge 2\textsc{lbs}(\sigma)]$



[6] For more on Navajo degree constructions, consult the paper by Coppock & Bogal-Allbritten in this volume and references therein.
[7] Svenonius & Kennedy (2006) report Norwegian *kor gammel* 'how old' formed with locative *kor*, as in *Kor er han*? 'Where is he?' – another spatial source for scalar morphemes. Hohaus (2018) charts the evolution of a directional particle ('forth, away') into a comparative marker in Samoan. English uses *way* and *far* as differentials (*far larger, way happier*).

In general, a measure phrase differential can be taken to be a predicate of type $\langle\sigma,t\rangle$ that describes the length of a segment. Given the conjunctive semantics and the rule of Segment Identification in (18), as far as the semantics is concerned, the measure phrase could be attached at various points in the structure below the final existential segment quantifier, '$\exists_\sigma$'. In Schwarzschild (2012), I provided evidence that in Hindi comparatives (at least those without *zyaadaa*), measure phrase differentials are attached to the adjective, which requires the adjective to be a segment predicate as in (14) above.

In this section, I've sketched a segmental analysis of simple phrasal comparatives. It was built around the idea that gradable adjectives are predicates of segments. In the next section, I'll propose a different kind of meaning for adjectives and we'll see how to reconcile the two ideas. The system of semantic types provides a handy way of marking progress. Here's the type system as it looks so far:

(24) Denotation domains

$$D_t = \{\text{TRUE, FALSE}\}$$
$$D_e = \{x : x \text{ is an individual}\}$$
$$D_d = \{d : d \text{ is a degree}\}$$
$$D_{\langle a,b\rangle} = \{f : f \text{ is a function from } D_a \text{ to } D_b\}$$
$$D_\sigma = \{\langle u,v,\mu\rangle \mid u \in D_d \wedge v \in D_d \wedge \mu \in D_{\langle e,d\rangle} \wedge u,v \text{ are in the range of } \mu\}$$

Within this system, gradable adjectives, measure phrases, and the comparative markers $-$*er* and *more* are all type $\langle\sigma,t\rangle$. The standard-marking preposition *than* is type $\langle e, \langle\sigma, t\rangle\rangle$ as is the thematic role head *End.*

3 Constructing degrees

As is customary in the degree semantic literature, up to now, I've presupposed the existence of a cornucopia of degrees, a different kind for each distinct kind of gradable predicate. And each variety has its own ordering. A tiredness degree is different from a height degree and they are not ordered with respect to one another. And in discussing measure phrases, I took for granted that it makes sense to talk about the *distance* between degrees of the same kind. My goal in this section is to assume an ontology based on individuals and possible worlds and to construct degrees out of those ingredients. Doing this will make for a sharper understanding of the nature of degrees and of implicit assumptions we make about them.[8]

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[8] Bale (2011) argued that degrees are derived from more basic relations between individuals. His argument was based on the effects of including *for* phrases in comparatives (*Esme is taller for a woman than Seymour is tall for a man*). Like Hoeksema (1983:423), Klein (1991:679) and Rullmann (1995:125), Bale equates Seymour's degree of height with the set of individuals whose height is *the same* as Seymour's. Schwarz (2010) challenged Bale's analysis with examples in which the *for* phrase was not local to the adjective (*Mia has a more <u>expensive</u> hat <u>for a 3-year old</u> than*

Relations

I am going to adopt a particular version of the idea that gradable predicates encode relations between individuals. On this version, the relation encoded by *heavy* holds between an object with weight and any other actual or possible object with equal or less weight. The various formulations in (25)-(28) below should serve to further illustrate the idea. Following Cresswell (1976:281), my meanings make use of entity-world pairs to be referred to as "possible individuals". I use two variables enclosed in angled brackets as a variable over such pairs. The reason for choosing a relational meaning of this particular type should become clear as we go along.

(25) $[\![heavy]\!]^{w,g,c} = \lambda\langle x,w'\rangle\ \lambda y.$ y in w is as heavy as, or heavier than x is in w'.

(26) $[\![expensive]\!]^{w,g,c} = \lambda\langle x,w'\rangle\ \lambda y.$ y's price in $w \geq x$'s price in w'.

(27) $[\![complicated]\!]^{w,g,c} = \lambda\langle x,w'\rangle\ \lambda y.$ y's level of complication in w meets or exceeds x*'s* level of complication in w'.

(28) $[\![remarkable]\!]^{w,g,c} = \lambda\langle x,w'\rangle\ \lambda y.$ y is remarkable in w to the same or greater degree as x is in w'.

These relations represent the ability we have to compare a given entity with others, real or imagined, in terms of weight, price, complexity, exceptionality and so on.

As Faller (2000:168) and Bale (2007, this volume) point out, individual relational meanings have an advantage over more familiar $\langle d,\langle e,t\rangle\rangle$ meanings like in (29)-(30) when it comes to conjunctions of adjectives like in the comparatives in (31)-(33).

(29) $[\![expensive]\!]^{w,g,c} = \lambda d.\ \lambda x.\ d$ is x's price in w.

(30) $[\![complicated]\!]^{w,g,c} = \lambda d.\ \lambda x.\ d$ is x's level of complication in w.

(31) This snake is more poisonous and aggressive than at least one of the others we examined.

(32) Hospital deaths are more expensive and intrusive than they once were.

(33) They made computing more expensive and complicated than we might have.

Interpreting *expensive and complicated* intersectively results in (34), on the proposed individual relational meaning, and it results in (35) assuming $\langle d,\langle e,t\rangle\rangle$ meanings as input.

(34) $\lambda\langle x,w'\rangle\ \lambda y.$ y's price in $w \geq x$'s price in w' and y's level of complication in w meets or exceeds x*'s* level of complication in w'.

(35) $\lambda d\ \lambda x.\ d$ is x's price in w and d is x's level of complication in w.

Sam does for a 9-year old). The construction of degrees described here differs from Bale's. It arose as a response to Schwarz's challenge (Schwarzschild 2013).

(34) relates any entity y that has a price and a level of complication with any possible individual whose price and level of complication is the same or less than that of y. (35) is hopeless, assuming that a level of complication can't also be a price[9]. In (31)-(33), I chose examples with quantifiers in the comparative clause. This rules out a possible conjunction reduction analysis. (31) for example is not equivalent to (36) below[10].

(36) This snake is more poisonous than at least one of the others we examined and this snake is more aggressive than at least one of the others we examined.

The meanings offered in (25)-(28) above do not capture selectional restrictions associated with those adjectives. *heavy,* for example, is restricted to applying to objects with weight and so in place of (25) we should have:

(37) $[\![heavy]\!]^{w,g,c} = \lambda\langle x,w'\rangle : x$ has weight in w'. $\lambda y : y$ has weight in w. y in w is as heavy as, or heavier than x is in w'.

It's important to keep selectional restrictions in mind in assessing the developments in the next section, nevertheless we'll revert to (25), the version of (37) that's been edited for space and clarity.

Degrees

Using '@' to stand for the actual world, the actual extensions of *heavy* and *complicated* are given in (38) and (39):

(38) $[\![heavy]\!]^{@,g,c} = \lambda\langle x,w'\rangle\ \lambda y.$ y's weight in @ $\geq$ x's weight in w'.

(39) $[\![complicated]\!]^{@,g,c} = \lambda\langle x,w'\rangle\ \lambda y.$ y's level of complication in @ meets or exceeds x's level of complication in w'.

A man who weighs 70 kilo is related by (38) to the set of possible individuals whose weight is 70 kilo or less. If Jack's taxes have a certain level of complication, (39) relates his taxes to the set of possible individuals with that same level or less. I propose that we

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[9] The problem in (35) can be avoided along the lines of how Champollion (2015) solves a related problem in event semantics. Instead of *walk* being a predicate of walking events, you treat it as a predicate of sets that include walking events, then the meanings of *walk* and *talk* can be combined intersectively. Similarly here, one could replace the degree argument of *expensive* in (29) with a set containing x's price. This is in fact how Dotlačil and Nouwen (2016) interpret degree predicates.

I am presupposing that *and* has a meaning that allows it to join two predicate meanings, a view that has come under fire recently in Schein (2017) and Hirsch (2017).

[10] See Bale (this volume) for further arguments against an ellipsis analysis.

identify these sets of possible individuals with degrees of heaviness and complication respectively and that in general:

(40) A **degree** is a set of individual-world pairs.

Under this proposal, degrees are ordered by the subset relation. If Jack weighs 70 kilo and Jill weighs 60 kilo, then the degree assigned to Jill is a subset of the degree assigned to Jack.

(41) $\{\langle x,w' \rangle \mid$ Jill in @ is as heavy or heavier than x is in $w'\} \subset \{\langle x,w' \rangle \mid$ Jack in @ is as heavy or heavier than x is in $w'\}$

Using 'd_{Jill}' to stand for Jill's actual degree of heaviness, we can record the fact in (41) compactly as:

(42) $d_{\text{Jill}} \subset d_{\text{Jack}}$

Subset imposes a partial ordering on degrees. It holds between heaviness degrees but not between a degree of heaviness and a degree of complication or between a degree of heaviness and a degree of tallness. To see this, observe that (41) above says that for any possible individual *u,* if *u*' weight is less than or equal to Jill's, then *u*'s weight is less than or equal to Jack's. Suppose now that Jill's *heavy* degree were ordered by subset below Jack's *tall* degree. In that case we would have:

(43) $\{\langle x,w' \rangle \mid$ Jill's weight meets or exceeds x's weight in $w'\} \subset \{\langle x,w' \rangle \mid$ Jack's height meets or exceeds x's height in $w'\}$

What (43) says is that for any possible individual *u,* if *u*'s weight is less than or equal to Jill's, then *u*'s height is less than or equal to Jack's. To say that is to say that it is not possible to weigh less than Jill but be taller than Jack. If Jill weighs 150lbs and Jack's height is 5 feet, then (43) entails that it is *logically impossible* for a person to be 6ft tall and weigh 140lbs. But that can't be, height and weight just aren't correlated in that way. Nor are weight and degree of complication or redness or temperature. Based on this observation we can define what it means for two degrees to be commensurate:

(44) d and d' are **commensurate** *iff* $d \subset d' \lor d' \subset d \lor d = d'$

Using that definition we can define the relation '<' that orders degrees:

(45) '$d < d'$' is defined when and only when d and d' are commensurate.

when defined: $d < d'$ *iff* $d \subset d'$

The ordering of degrees reflects the underlying ordering of individuals. If Jack is older than Jill, then Jack's *old*-degree is ordered above Jill's. In the previous section, we used the symbol '$\nearrow$' to mean a rising segment. We can now define it more precisely[11]:

(46) $\nearrow \stackrel{\text{def}}{=} \lambda\sigma\ \text{START}(\sigma) < \text{END}(\sigma)$

In discussions of the semantics of comparatives and other degree constructions, one finds locutions such as "the degree to which the plane is high", "the extent to which Jack is funny", "John's degree of sloppiness", "John's coldness degree" or "Joe's degree of intelligence". These expressions presuppose a unique degree that a gradable adjective associates with an individual. Here's a recipe that underwrites this presupposition[12]:

(47) For any gradable predicate α and individual z, $\boldsymbol{\{\langle x, w'\rangle \mid [\![\alpha]\!]^{w,g,c}(\langle x,w'\rangle)(z)\}}$ is "the degree to which z is α in w" or "z's degree of α-ness in w"

EXAMPLE Joe's actual degree of intelligence is:

a. $\{\langle x, w'\rangle \mid [\![\textit{intelligent}]\!]^{@,g,c}(\langle x,w'\rangle)(\textit{joe})\}$

b. $\{\langle x, w'\rangle$ | Joe is of equal or greater intelligence in @ as x is in $w'\}$

c. the set of possible individuals whose intelligence Joe's meets or exceeds.

Let's now explore some of the consequences of this conception of a degree. It turns out that on the proposed construction, Jack's degree of tallness is incommensurate with his degree of shortness. The incommensurability of tallness and shortness degrees is a building block of accounts of cross-polar anomaly (Seuren 1978, von Stechow 1984b, Kennedy 2001).

If a glass is full, every other possible container will be as full or less full. None will be more full. In that case, the glass has the maximal degree of fullness: the set of all possible containers. Not all adjectives give rise to maximal degrees. The distinction between those that do and those that don't is reflected in the distribution of proportional modifiers such as *mostly, completely* and *slightly* and it plays a role in the interpretation of the positive (Rotstein and Winter 2004, Kennedy & McNally 2005, 2010, Kennedy 2007a, for discussion and further references see Morzycki 2015:§3.7.2).

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[11] Incorporating commensurability as a definedness condition yields:

$\nearrow \stackrel{\text{def}}{=} \lambda\sigma : \text{START}(\sigma)$ and $\text{END}(\sigma)$ are commensurate. $\text{START}(\sigma) \subset \text{END}(\sigma)$

[12] Degreehood is defined in a structurally similar way in Kamp & Partee (1995:153). Unfortunately, I haven't studied the recent work by Heather Burnett, Jenny Doetjes, Robert van Rooij and others building on Kamp (1975) and Klein (1980), but I suspect there is much there that parallels what I am doing here.

There is a standard way of assigning numbers to extended objects so that mathematical facts about the numbers reflect length-related facts about the objects. The sum of the lengths of two poles each expressed in numbers of meters will equal the length in meters of the result of placing them together **end to end**. Nothing of this sort has been done for *remarkability*. Whether it is even possible to do that is a question for measurement theory. In (25)-(28), I included the adjectives *complicated* and *remarkable* to emphasize the fact that although degrees might sometimes be associated with expressions from established measurement systems, the existence of a system of measurement is not a prerequisite for defining degrees.[13] Still, we do use expressions that come from measurement systems in degree constructions. It is worth dwelling on some such uses to clarify how and where we rely on the measurement system to achieve the desired meaning. In (48), the phrase *10ºC* is used as a differential, which in §2 was taken to mean it is a predicate of scale segments describing the distance between two degrees. This is indicated in the bolded conjunct in (49).

(48) The tea is 10ºC hotter than the milk.

(49) $\exists\sigma\ [\text{END}(\sigma) = \mu_\sigma(\text{Tea}) \wedge \mathbf{10^{o}\boldsymbol{C}(\sigma)} \wedge \mu_\sigma = \text{HEAT} \wedge \nearrow(\sigma) \wedge \text{START}(\sigma) = \mu_\sigma(\text{Milk})]$

In order to develop an idea about what '$10^{o}C(\sigma)$' says, it will help to first bring the discussion of comparatives like (48) in line with our new meanings for adjectives.

We begin by observing that the function named 'HEAT_w' defined below in (50) is in fact a measure function. To each individual *y*, it assigns a set of possible individuals, which, by the construction in (47) is the degree to which *y* is hot in *w*[14].

(50) $\text{HEAT}_w \stackrel{\text{def}}{=} \lambda y.\ \{\langle x,w'\rangle \mid [\![hot]\!]^{w,g,c}(\langle x,w'\rangle)(y)\}$

$\text{HEAT}_w \stackrel{\text{def}}{=} \lambda y.\ \{\langle x,w'\rangle \mid y\text{'s temperature in } w \geq x\text{'s temperature in } w'\}$

In the previous section we took gradable adjectives to be predicates of segments that established the measure function of the segment, for example:

(51) $[\![tall]\!]^{g,c} = \lambda\sigma.\ \mu_\sigma = \text{HEIGHT}$

To get to that kind of meaning in the new context we'll make use of an operator that combines with an adjective denoting a relation among individuals and yields a predicate of segments. We'll realize that operator as a symbol composed of an '*S'* for scale and a line through it representing a segment. The operator is defined in (52) below using '*R*' as

[13] Sassoon (2010:176-7) agrees on this point even though, as that paper's title suggests, measurement is the foundation on which a degree semantics is built and conventional measurement systems, where they do exist, play a role in the interpretation of the relevant adjectives.

[14] Strictly speaking, the meaning of *hot* is a function which is restricted to those entities that have a temperature and that selectional restriction is inherited by HEAT_w.

a variable over gradable adjective meanings. The label '$e{\times}s$' is the type of individual-world pairs – possible individuals:

(52) $[\![\$]\!]^{w,g,c} = \lambda R_{\langle e\times s,\langle e,t\rangle\rangle}\lambda\sigma.\ \mu_\sigma = \lambda y.\ \{\langle x,w'\rangle \mid R(\langle x,w'\rangle)(y)\}$

Combining that operator with *hot* we get:

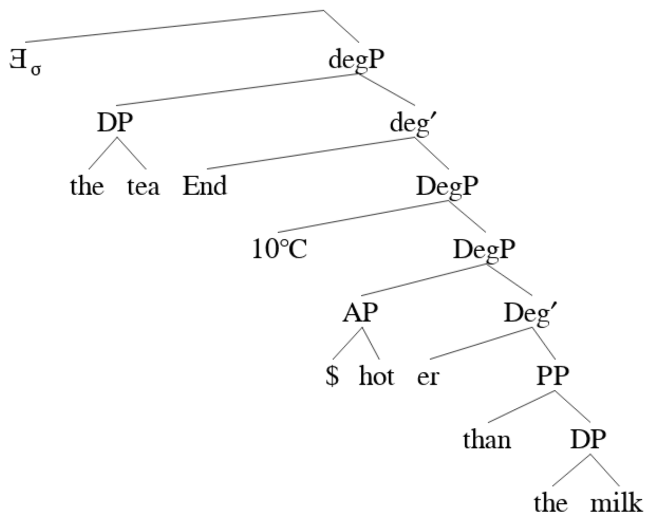
(53) $[\![\$\ hot]\!]^{w,g,c} = \lambda\sigma.\ \mu_\sigma = \lambda y.\ \{\langle x,w'\rangle \mid [\![hot]\!]^{w,g,c}(\langle x,w'\rangle)(y)\}$

Using (50), we have:

(54) $[\![\$\ hot]\!]^{w,g,c} = \lambda\sigma.\ \mu_\sigma = \mathrm{HEAT}_w$

Using our new *$* operator, we have the following structure for (48):

(55)



(56) $\exists\sigma\ [\mathrm{END}(\sigma) = \mu_\sigma(\text{the.tea}) \wedge 10^{o}C(\sigma) \wedge \mu_\sigma = \mathrm{HEAT}_w \wedge \nearrow(\sigma) \wedge \mathrm{START}(\sigma) = \mu_\sigma(\text{the.milk})]$

Our interest lies now in the conjunct '$10^{o}C(\sigma)$'. We want to explain how a phrase, *10°C*, whose meaning relies on a particular system of measurement can be used to say something about a triple consisting of two degrees (sets of possible individuals) and a measure function. The analysis provided below in (57)-(59) is designed to capture two intuitions behind the use of the statement '$10^{o}C(\sigma)$': (a) the measure function in σ tracks those properties that the Celsius measurement system is designed to reflect and (b) any two individuals assigned the two degrees in σ will have Celsius temperatures whose scalar quantities differ by 10.

(57) Say that a measure function μ **correlates** with Celsius if the following holds:
Every entity in the domain of μ has a temperature.
For any *x,y* in the domain of μ, if the temperature of *x* on the Celsius scale is greater than that of *y*, then $\mu(y) < \mu(x)$

(58) Let CELSIUS be a partial function from *degrees* to *numbers* such that for any μ that correlates with Celsius, for any object x in the domain of μ, $\text{CELSIUS}(\mu(x)) = n$ *iff* the temperature of x is nºC

(59) $[\![10^{o}C]\!]^{w,g,c} =$

$$\lambda\sigma : \mu_{\sigma} \text{ correlates with Celsius. } |\text{CELSIUS}(\text{START}(\sigma)) - \text{CELSIUS}(\text{END}(\sigma))| = 10$$

'$|n-m|$' stands for the absolute value of $n - m$. Absolute value is needed here because the start of a segment could be higher or lower than the end. (48) describes a segment that begins with the milk's degree of heat and ends with the tea's degree of heat. And those degrees correspond in the sense of (58) to Celsius temperatures that differ by 10. In this analysis, degrees are not added or subtracted. Measurements are not added or subtracted either. Temperature is not like length in which addition and subtraction of measurements is meaningful. The subtraction in (59) is purely numerical. As an aside, the appeal to absolute value makes the welcome prediction that measure phrases formed with negative numbers can't serve as differentials (*-5^{o} *colder*, *-2^{o} *less hot*).

Nonce differentials like *a boxcar* in (60) below do not rely on a conventional system of measurement, so when we interpret them we rely on the context and on the world of evaluation to fill in the details of what counts as a standard and what the measurement procedure is.

(60) The new engine is a boxcar longer than the old engine.

a boxcar as used here describes a scale segment; as such it describes the distance between two degrees. If d_1 and d_2 are *long*-degrees and $d_1 < d_2$, then d_1 and d_2 "differ by a boxcar" as long as the following holds: For any object o whose *long*-degree is d_1, if o is concatenated with a boxcar, the result is an object having *long*-degree d_2. Exactly what that amounts to crucially relies on how long a boxcar is and the truth conditions of (60) will differ accordingly. In effect, a kind of contingent measurement system for length is assumed in which a boxcar is the standard and that measurement system plays the role that is played by Celsius in the analysis above.

Differential *a lot* is used in comparatives based on gradable adjectives of any variety. It does not rely on a particular system of measurement. In (62) below I offer a definition with the following idea in mind. If Carla is a lot nicer than Bob, then the difference between Carla and Bob's degrees of niceness is relatively large compared with Bob's degree. Like degrees, degree-differences are sets of possible individuals, so we'll need to make use of a contextually supplied measure on sets[15], notated m_c, to assign a size to the difference. We'll make use of the following observations:

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[15] A *measure* is a way to talk about the relative 'size' of infinite sets. It's a function from sets to numbers. A measure needs to fulfill certain requirements, among them, for sets A, B, C in its domain, if $A \subset B$, then $m(A)$ is a smaller number than $m(B)$. If $(A \cap C) = \emptyset$, then $m(A)+m(C) = m(A \cup C)$. A *measure* is different from a *measure function*. The latter assigns degrees. I would have preferred the term "degree function", but that has

(61) $(\textsc{start}(\sigma) \cup \textsc{end}(\sigma))$ is the larger of the two degrees making up σ.

$(\textsc{start}(\sigma) \cap \textsc{end}(\sigma))$ is the smaller of the two degrees making up σ.

$((\textsc{start}(\sigma) \cup \textsc{end}(\sigma)) - (\textsc{start}(\sigma) \cap \textsc{end}(\sigma)))$ is the difference between the larger and the smaller of the two degrees making up σ. It includes those possible individuals in the higher degree not in the lower one (if the degrees are equal this will be ∅).

(62) $[\![a\ lot]\!]^{w,g,c}(\sigma) = 1$ *iff* $m_c((\textsc{start}(\sigma) \cup \textsc{end}(\sigma)) - (\textsc{start}(\sigma) \cap \textsc{end}(\sigma))) > p_c(m_c((\textsc{start}(\sigma) \cap \textsc{end}(\sigma))))$

m_c is a contextually supplied measure on sets.

p_c is a contextually supplied proportion.

A is a lot taller than B says that the set of possible individuals that are taller than B but not taller than A is big relative to the degree to which B is tall. A difference of 1-foot might be sufficient to be a lot taller than a person but insufficient to count as a lot taller than a building.[16]

In addition to serving as differentials, measure phrases are employed on a limited basis as adjectival modifiers (Doetjes 2012, Faller 2000, Sawada & Grano 2011, Schwarzschild 2005, Svenonius & Kennedy 2006, Winter 2005). Differential and modifier uses cannot be reduced one to the other. In German, as in English, measure phrases formed with negative numbers cannot be used as differentials (* *−5º Kälter* '−5ºcolder'), but they can modify adjectives (*−5º Kalt* '−5º cold'). Going the other way, in English positive *5º* cannot be used as an adjectival modifier (*5º *cold*), but it can serve as a differential (5º *colder*). To facilitate the combination of a measure phrase and an adjective, Svenonius & Kennedy (2006:105), following the logic of Kennedy (1999), posit a functional head 'Meas' that combines with an adjective to form a predicate of type $\langle d,\langle e,t\rangle\rangle$. *Meas* selects for *tall* and *old* but not for *heavy* and *cold*. Implementing this idea in the present system, gives us:

(63) $[\![Meas]\!]^{w,g,c} = \lambda R_{\langle e\times s,\langle e,t\rangle\rangle}\ \lambda d\ \lambda y.\ \ d = \{\langle x,w'\rangle \mid R(\langle x,w'\rangle)(y)\}$

This proposal presupposes that measure phrases themselves can name degrees. *Six feet* must be taken to stand for the degree of height possessed by those individuals whose length when correctly measured in feet in the vertical direction yields six. 5º*F* indirectly

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been used to refer to meanings of modifiers like *quite* and *very*, moreover "measure function" is the standard term in the semantics literature for functions whose range is degrees.

[16] Differential *much* adds another layer of complexity, since it is a gradable predicate and so it has its own type $e\times s$ argument. For discussion of *much* in various contexts, see Rett (2018).

names the degree of coldness possessed by individuals whose Fahrenheit temperature is 5º. This nomenclature is familiar. It is usually employed without comment in the degree semantics literature.

This completes our discussion of degrees. The core idea is to conceive of degrees as sets of possible individuals and to introduce degrees, segments and measurements through functional heads that combine with gradable predicates. The segment introducing operator '*$*', defined in (52), is one of those functional heads as is the degree introducing '*Meas*' in (63). In §5 below, the comparative marker *more/–er* that occurs in clausal comparatives will also be analyzed as a segment introducer.

Taking degrees to be sets allows us to identify the partial order that holds of degrees as the subset relation. Taking them to be sets of *logically possible* individuals, allows for comparison across worlds. The degree to which Terry is successful is ordered with respect to the degree to which he would have been successful had Charley looked out for him. We've also made clear how locutions whose interpretation flows from systems of measurement interact with degree talk without making measurement a precondition for degree talk. And we've discussed how mass quantifiers like *a lot* function as differentials. Deriving degrees in the syntax means that gradable adjectives are predicates of individuals and this allows for an account of adjective conjunctions ((31)-(34)) and it allows for a better account of *for* phrases in comparatives (footnote 8).

In (64) below, I've summarized the discussion to this point in the form of an assignment of denotation domains to semantic types. Up to now, I've been talking about degrees as sets of individuals because it's easier to think about them that way. But in carrying out a compositional semantics, it's better to take them to be the functions that characterize those sets, functions in the domain of type $\langle e{\times}s, t\rangle$.

(64) Denotation domains

$D_t = \{\text{TRUE, FALSE}\}$

D_e = set of individuals.

D_s = set of possible worlds.

$D_{\langle a,b\rangle} = (D_a \rightarrow D_b)$

$D_{e\times s} = \{\langle x,w'\rangle \mid x \in D_e \wedge w' \in D_s\}$ (possible individuals)

$D_d = D_{\langle e\times s,\, t\rangle}$ (degrees)

$D_\sigma = \{\langle u,v,\mu\rangle \mid u \in D_d \wedge v \in D_d \wedge \mu \in D_{\langle e,d\rangle} \wedge u,v \in \text{Range}(\mu)\}$ (segments)

4 Typological landscape

In the previous two sections, I fleshed out the two hypotheses under investigation, repeated below:

(a) Degree constructions make use of quantification over scalar segments, parts of a scale.

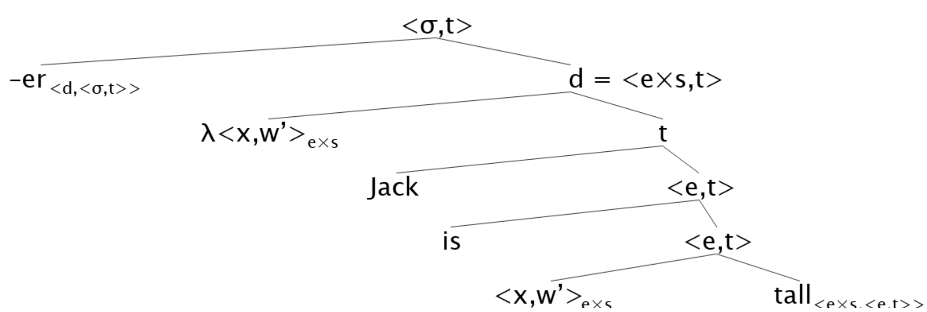
(b) Gradable predicates denote relations between possible individuals. Degrees and segments are introduced with the functional vocabulary. They are defined in terms of possible individuals.

If theses (a) and (b) are universally valid, then any language that has a degree construction will perforce have an overt or covert operator such as *$* or *Meas.* The presence of measure phrase modifiers and comparatives in Q'eqchi' exhibited in (7)-(8) above therefore implicates a null *$* and *Meas* in that language. By contrast, Bochnak (2015) demonstrates the absence of degree constructions in Washo, which implies that it does without degree or segment introducing operators (Lassiter 2015:153). Bochnak argues that degrees as such are not part of the basic ontology of Washo, but he concludes (p 41) with an argument for a thesis like in (b) "under my analysis, there are languages that simply lack a basic semantic type, namely degrees. This raises the question of why degrees should be subject to this kind of cross-linguistic variation. It is much less obvious that other logical types should be missing from a language (e.g., individuals, events, worlds), or what a language would look like if such a gap were to exist. I speculate that this point can be linked to the idea that degrees are not in fact basic on a par with other simple types. … if degrees don't come 'for free' as basic elements in the model, then languages differ on whether they choose to derive them."

The *$* morpheme was posited as a way to introduce segments when gradable predicates have type $e{\times}s$ arguments. A different option, to be explored in the next section, takes the comparative marker (*er*/*more*) to be of type $\langle d,\langle \sigma,t\rangle\rangle$:

(65) $[\![-er]\!] = \lambda d\ \lambda\sigma\ \text{END}(\sigma) = d$

When this morpheme combines with a type $\langle e{\times}s,\langle e,t\rangle\rangle$ predicate, there is a type mismatch which can be resolved through quantifier raising. *–er* is looking for an argument of type *d,* i.e. $\langle e{\times}s,t\rangle$, so when it raises it leaves a trace of type $e{\times}s$ (compare, a generalized quantifier looks for an argument of type $\langle e,t\rangle$, so when it raises, it leaves a trace of type *e*):



Suppose then that English makes do with *–er* and does not have a null *$* operator. In that case, we have languages with null segment operators (Q'eqchi'), languages with overt segment operators (English) and languages with no such operators (Washo). A fourth possibility is a language that has both null and overt segment operators. In such a language, unlike in English, the overt comparative marker would be optional. Hebrew and Hindi are two such languages (Schwarzschild 2010, 2012). Both have an overt comparative marker but can form comparatives without it, although only with certain adjectives. In those languages, the null segment operator is selective, like the *Meas* operator discussed above.

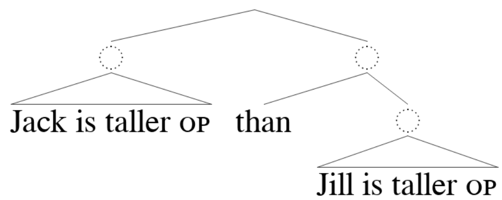
Another parameter of variation that emerges from discussion to this point is the form and interpretation of the standard marker, the morpheme that introduces the START of the comparative. In the languages canvassed, START is indicated with a case-marker (Greenlandic), a preposition (Q'eqchi') and a postposition (Navajo). This task can also be performed by a verb (e.g. *surpass*) and in some cases by a particle derived from a conjunction (Stassen 2006). Comparatives formed in this way display syntactic properties characteristic of coordinations. Stassen points to gapping in Dutch. Lechner & Corver (2017:§4) discuss a range of such effects in English. Along these lines, in the following section, *than* will be understood to combine two identically constructed clauses.

A final parameter of variation is the locus of the ↗ operator. Recall, that in Navajo, it is unquestionably packaged together with the standard marker. Kennedy (2007b) argues that this is the norm cross-linguistically (see also Menon 2017). We'll adopt that proposal in the following section.

5 Clausal Comparatives

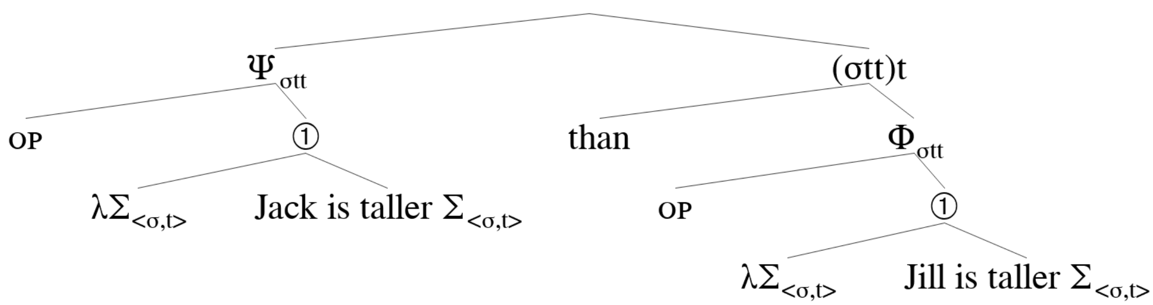
von Stechow (1984a:55) treated comparative clauses as predicates of degrees in the scope of a maximality operator. This architecture successfully captured the meaning of comparative clauses containing the modal *can* and negative polarity *any,* and the semantics underwrites the claim that comparative clauses allow negative polarity items because they are downward entailing (von Stechow 1984a:70, Rullmann 1995:§2.4). In Schwarzschild & Wilkinson (2002), we proposed a semantics based on intervals (sets of degrees) which captured the meaning of comparative clauses containing quantifiers that are not negative polarity items. We also showed with examples using those quantifiers that comparative clauses are upward entailing. The two papers covered different data and were incompatible. Heim (2000, 2006) explained away the apparent contradiction by positing an operator inside the comparative clause that transitions from degrees to intervals. The environment below the operator is downward entailing. *can, have-to* and negative polarity items are situated there. The environment above the operator is upward entailing and non-NPI quantifiers are located there along with propositional attitude verbs and some modals. In this section, I hope to show that the bifurcation of the comparative clause that Heim discovered stems from the need to transition from predicates of possible individuals to predicates of segments.

Following Alrenga, Kennedy, and Merchant (2012), I take *than* to be a contentful expression that combines with two clauses both of which are formed with a comparative marker and a silent operator:



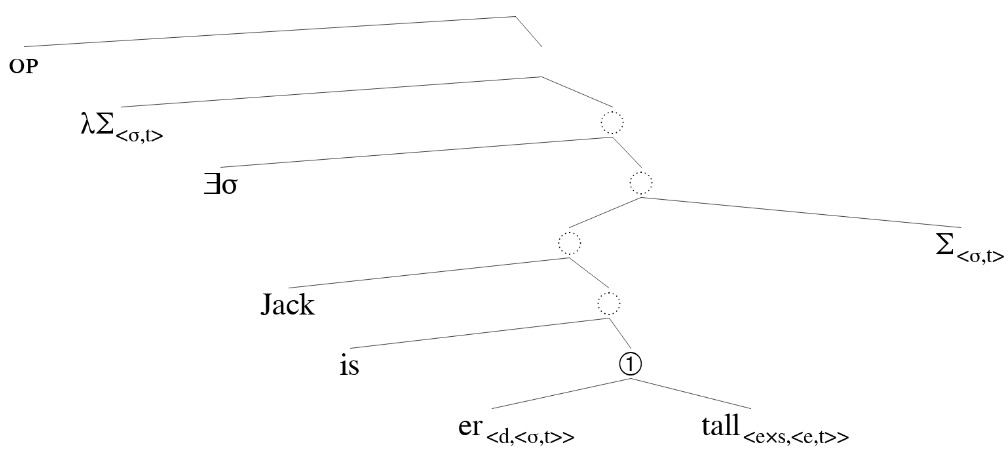
Jack is taller than Jill is

Izvorski (1995:15) suggests that the empty operator in comparatives is an adverbial amount/degree WH-expression. Adapting that suggestion to the current context, I take the operator to range over segmental modifiers and to therefore leave a trace of type $\langle\sigma,t\rangle$[17]. In syntactic structures, I'll use a Σ for type $\langle\sigma,t\rangle$ traces. The operator itself denotes the identity function, so the denotations for the nodes labeled ① is passed up to the dominating nodes, Ψ and Φ:



Jack is taller than Jill is

Turning now to the formation of the higher clause Ψ, we start with the structure below which features the segment existential '$\exists\sigma$' from section 2:



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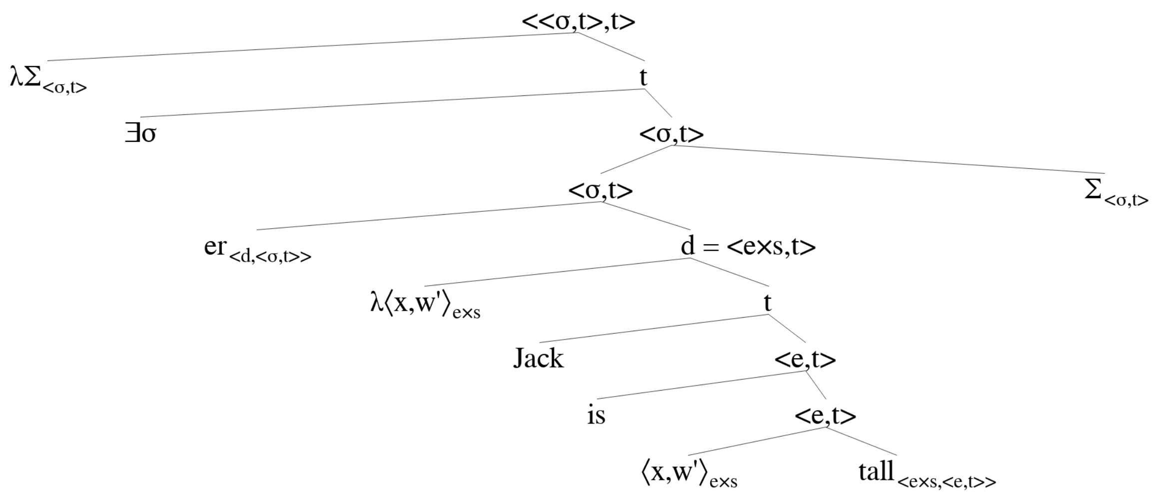
[17] $[\![\text{OP}]\!]^{w,g,c} = \lambda\psi_{\langle\langle\sigma,t\rangle,t\rangle}.\psi$. Being type $\langle\langle\langle\sigma,t\rangle,t\rangle t\rangle$, OP leaves a trace of type $\langle\sigma,t\rangle$, by analogy with WH expressions of type $\langle\langle e,t\rangle,t\rangle$ that leave type e traces. Based on a suggestion of Roger Higgins', Grimshaw (1987:668) posited "a phonologically null Adverb Phrase, something like *to a certain/great extent*, within the subcomparative clause". Izvorski adopted this idea and further proposed that this null phrase undergoes WH-movement.

There is a type mismatch at ① which is resolved by raising *–er* and leaving a trace of type $e{\times}s$.



Given the definition in (47), the phrase labeled *d* denotes the degree to which Jack is tall, in other words, his height[18]. Given the meaning for *–er* introduced earlier,

(66) $[\![-er]\!]^{w,g,c} = \lambda d.\ \lambda\sigma.\ \textsc{end}(\sigma) = d$

the lower phrase labeled $\langle \sigma,t\rangle$ will have the meaning in (67) and the higher phrase labeled $\langle \sigma,t\rangle$ will have the meaning in (68) ('Σ' is used for traces in the object language and is the corresponding variable in the metalanguage)

(67) $\lambda\sigma\ \textsc{end}(\sigma) = \text{Jack's height}$

(68) $\lambda\sigma\ [\textsc{end}(\sigma) = \text{Jack's height} \wedge \Sigma(\sigma)]$

The entire clause denotes a predicate of sets of segments:

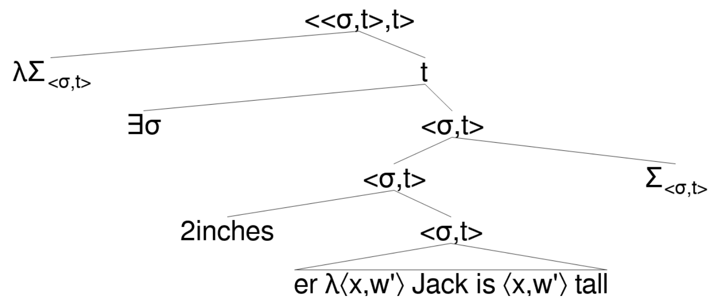
(69) $\lambda\Sigma\ \exists\sigma\ \textsc{end}(\sigma) = \text{Jack's height} \wedge \Sigma(\sigma)$

The clause under *than* labeled 'Φ' above is formed in the exact same way, and parallel to (69), if it is true of a set, then that set includes a segment ending in Jill's height.

As discussed in §2 and 3, differential measure phrases are type $\langle \sigma,t\rangle$ expressions. As such, they can be adjoined immediately above or below the WH operator:

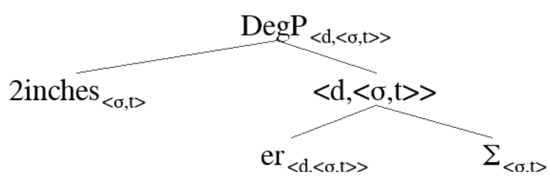
[18] More precisely, Jack's height in the world of evaluation. In building up the theory, I've suppressed reference to the world of evaluation. The metalanguage 'height' in (68), (69) and subsequent formulas should be 'height_w'.

(70)



An alternative syntax, which I will not pursue here, keeps the differential and the WH operator inside the DegP. It would give us the structure in (71) below, interpretable through two applications of the rule of Segment Identification in (18) which would yield the meaning '$\lambda d\ \lambda\sigma\ 2\text{INS}(\sigma) \wedge \text{END}(\sigma) = d \wedge \Sigma(\sigma)$'.

(71)



Summarizing now, the meaning of the comparative in (72) is arrived at by applying the meaning of *than* to the meanings in (73) and (74):

(72) Jack is 2 inches taller $\boxed{\text{than}}$ Jill is.

(73) $[\![\text{OP}\ \lambda_2\ \exists\sigma\ \ 2\text{ins}\ [-er\ \lambda_1\ \text{Jack is}\ t_1\ \text{tall}]\ t_2]\!]^{w,g,c} =$

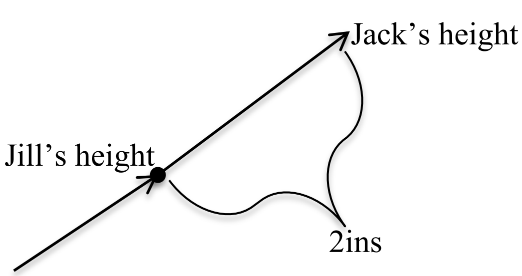
$$\lambda\Sigma\ \exists\sigma\ 2\text{INS}(\sigma) \wedge \text{END}(\sigma) = \text{Jack's height} \wedge \Sigma(\sigma)$$

(74) $[\![\text{OP}\ \lambda_2\ \exists\sigma\ [-er\ \ \lambda_1\ \ \text{Jill is}\ t_1\ \text{tall}]\ t_2]\!]^{w,g,c} =$

$$\lambda\Sigma\ \exists\sigma\ \text{END}(\sigma) = \text{Jill's height} \wedge \Sigma(\sigma)$$

The meanings in (73) and (74) both apply to a set of segments. If (73) is true of a set, that set includes a segment that ends with Jack's height and is 2inches long. If (74) is true of a set, that set includes a segment that ends with Jill's height.

The comparative in (72) is true under the circumstances illustrated below, in which a 2inch rising segment ending in Jack's height starts with the end of a segment ending in Jill's height.



Jack is 2 inches taller than Jill is

What remains now is to arrive at a meaning for *than* that requires what is illustrated above in terms of the meanings in (73) and (74).

The sets that (73) and (74) pick out will contain many segments that are of no interest to us, having nothing to do with Jack or Jill. Since (74) merely requires that Σ contain a segment ending in Jill's height, it could hold of a set containing a segment ending in Jill's height along with another segment ending in the weight of the moon. For this reason, we'll makes use of the *min* operator in (75) below adapted from Beck (2010, 2014) and Dotlačil & Nouwen (2016).

(75) $min(\Sigma_{\langle\sigma,t\rangle}, \Phi_{\langle\sigma t,t\rangle})$ *iff* $\Phi(\Sigma) \wedge \neg\exists\Sigma' [\Phi(\Sigma') \wedge \Sigma' \subset \Sigma]$

With (75), we can pick out the smallest sets that satisfy the clauses of the comparative. Sets satisfying (73) all contain at least one segment ending in Jack's height. A set that minimally satisfies (73) contains just one segment and that segment ends in Jack's height. Let's call such a segment, a *witness* for (73). Each set that minimally satisfies (73) contains such a witness. Each set that minimally satisfies (74) will contain a witness for (74), a segment that ends in Jill's height. What we want to require is that there be a witness ending in Jack's height that is rising and whose start is the end of a witness ending in Jill's height. The meaning for *than* in (76) below imposes that requirement ('Φ' corresponds to the meaning in (74) of *Jill is taller*):

(76) $[\![than]\!]^{w,g,c} = \lambda\Phi_{\langle\sigma t,t\rangle}. \lambda\Psi_{\langle\sigma t,t\rangle}.$
$\exists\Sigma_1\, min(\Sigma_1, \Phi)$
$\wedge \forall\sigma_1 \in \Sigma_1 [\exists\Sigma_2 [min(\Sigma_2, \Psi) \wedge \exists\sigma_2 \in \Sigma_2 [\nearrow(\sigma_2) \wedge \textsc{end}(\sigma_1) = \textsc{start}(\sigma_2)]]]$

Since there is a differential in *Jack is 2 inches taller than Jill is*, all the witnesses for the main clause will be the same length, assuming '2' is construed as 'exactly 2'. If there were no differential, the witnesses could differ in length. In that case, one could start with Jill's height, but others wouldn't. Because of the existential '$\exists\Sigma_2$', (76) correctly requires only that one of the witnesses start with Jill's height and end with Jack's.

Any set that minimally satisfies (74) will contain just one witness, just one segment ending in Jill's height, so the universal quantifier '$\forall\sigma_1$' would appear to be superfluous. It becomes important once we turn to comparative clauses with individual or world quantifiers in them:

(77) a. Jumpy is more slippery than every other fish is.
b. than OP λ_2 every other fish λx [$\exists\sigma$ [*–er* λ_1 *x* is t_1 slippery] t_2]
c. $\lambda\Sigma\, \forall z$ fish(z) $\rightarrow \exists\sigma$ END(σ) = z's slipperiness $\wedge\, \Sigma(\sigma)$

(78) a. Jumpy is longer than I expected he would be.
b. than OP λ_2 I expected [$\exists\sigma$ [*–er* λ_1 he would be t_1 long] t_2]

A set that minimally satisfies the comparative clause meaning in (77)c will contain, for each fish other than Jumpy, a segment ending in that fish's degree of slipperiness. (76) requires that for each one of those segments, there be a rising segment that ends in

Jumpy's degree of slipperiness and that starts with the fish's degree. That means that for each fish, Jumpy is more slippery.

A set that minimally satisfies the clausal complement of *than* in (78)b will contain, for each world compatible with what I expected, a segment ending in Jumpy's length in that world. If my expectation covered a range, say from 2 to 4 inches long, then that minimal set will have segments ending in degrees ranging from 2 to 4 inches. (76) requires that for each one of those segments, there be a rising segment ending in Jumpy's actual length that starts with the expected length. That means Jumpy's length has to exceed my expectation: he must be more than 4 inches long.

In (72), there is a differential in the main clause which determines the lengths of the witness segments. The comparative clause is also given a segmental semantics, so differentials are interpretable there as well. This possibility is realized in the following example repeated from the introduction:

(79) Jack and Jill are train enthusiasts. They've been discussing a high-speed freight train planned for their region. They wonder about whether the boxcars will be 60ft long, like on the Santa Fe line, or 50ft long, like on the Caroliner. Jack and Jill disagree about the engine size. Jack's expectation is that the engine will be 2 boxcars long. Jill expects it to be one boxcar long:

Jack expects the engine to be one boxcar longer than Jill does.

(80) a. Jack expects the engine to be a boxcar longer than Jill does Δ.
b. OP λ_2 Jack expects [$\exists\sigma$ boxcar [*–er* λ_1 the engine to be t_1 long] t_2]
c. than OP λ_2 Jill expects [$\exists\sigma$ boxcar [*–er* λ_1 the engine to be t_1 long] t_2]
d. Δ = expects $\exists\sigma$ boxcar [*–er* λ_1 the engine to be t_1 long] t_2

The differential measure phrase in (80)a needs to be interpreted under the scope of *expect*: there is no actual length of a boxcar at issue here, only expected lengths. Since the ellipsis in the comparative clause includes *expect,* it must include the differential as well. So let's see how this works out given the information reported in (79). Consider a set Σ_{Jill} minimally satisfying the clausal complement of *than* in (80)c. For each world compatible with Jill's expectation, there's a segment in Σ_{Jill} that is the length of a boxcar in that world (50ft or 60ft) and that ends with the length of an engine in that world (50ft or 60ft). So, there are two witnesses for Jill's expectation and they are depicted below as the lower arrows. Now, consider a set Σ_{Jack} minimally satisfying (80)b. For each world compatible with Jack's expectation, there's a segment in Σ_{Jack} that is the length of a boxcar in that world (50ft or 60ft) and that ends with the length of an engine in that world (100ft or 120ft). So Σ_{Jack} provides two witnesses for Jack's expectation depicted below as the upper arrows. As the meaning of *than* requires, for each witness σ_{Jill} in Σ_{Jill}, there is witness σ_{Jack} in Σ_{Jack} such that σ_{Jack} starts with the end of σ_{Jill}.[19]

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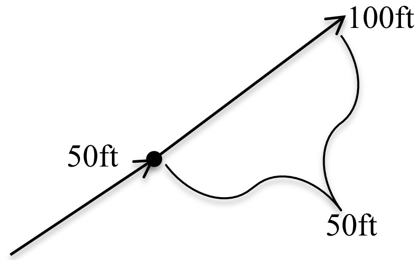
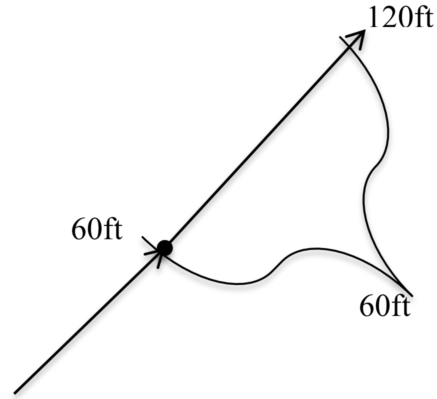
[19] This example has universal modal quantifiers in the two clauses joined by *than.* The example cited in Dotlačil & Nouwen (2016:64) and repeated below has universal individual quantifiers in the two clauses joined by *than*.





In (80)d, the material elided in (80)a is shown to include a differential, *boxcar*. Another kind of example that requires elided measure phrases in the comparative clause involve conjunctions of compared adjectives:

(81) This rod is 2 lbs heavier and 1inch longer than one of the tubes was.

Across the Board movement of the WH operator and of the subjects of both clauses produces these structures:

(82) a. OP λ_2 [The rod] λ_5 $\exists\sigma$ [2lbs [*–er* λ_1 x_5 is t_1 heavy] t_2] and $\exists\sigma$ [1inch [*–er* λ_1 x_5 is t_1 long] t_2]
b. than OP λ_2 [one of the tubes] λ_6 $\exists\sigma$ [2lbs [*–er* λ_1 x_6 is t_1 heavy] t_2] and $\exists\sigma$ [1inch [*–er* λ_1 x_6 is t_1 long] t_2]

Consider a set Σ_{tube} minimally satisfying the clausal complement of *than* in (82)b. It will include two segments, one with weights on either end and one with lengths on either end. This much follows from the meanings discussed in this section as well as the definition of a segment (64) and the constraint on measure functions imposed by measure phrases (59). There will be a different minimal set of segments for each one of the tubes (assuming they differ one from the other in weight or length). A set minimally satisfying the main clause will contain two witnesses, a weight segment and length segment. The meaning of

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(i) The state economies of Ireland, the Netherlands and Australia **all** scored higher than they **each** scored in the mid-1980s.

Their example reveals a potential lacuna. Our meaning for *than* requires that for every 1980s score of one of the countries, there is a higher current score of one of the countries. This needs to be strengthened perhaps pragmatically to say "there is a higher current score of that same country" or more generally there is a corresponding score that is higher. Likewise, there is a sense in the build up to our (79) of a correspondence between Jack and Jill's expectations.

than requires that we find one of the sets for the comparative clause (see '$\exists\Sigma_1$' in (76)) and relate both of its witnesses to the witnesses of the main clause. This will entail that:

(83) For one of the tubes, x_6, the rod is 2lbs heavier than x_6 and the rod is 1inch longer than x_6.

A possible verifying scenario is one in which the rod weighs 3lbs and is 5 inches long, while one of the tubes weighs 1lb and is 4 inches long. Let's call that tube x_6. Given that tube x_6 weighs 1lb, a segment that ends in its weight could be 2lbs-long, as (82)b requires, only if that segment is falling. That is possible. Although I've been drawing pictures in which the comparative clause segments are rising, nothing requires that. The rising predicate '↗' is in the meaning of *than,* and not in the meaning of *–er*.[20] In both (80) and (81), a differential was found to be present in the comparative clause. Were it not there, ellipsis would not have been possible. Given the way the meaning of *than* is formulated, however, the differential has no truth conditional effect inside the comparative clause. So unless it's facilitating ellipsis, I assume its presence would violate a constraint that punishes verbosity[21].

The conjunction in (81) has scope over the comparative, but, as anticipated in §3, the reverse is possible. Consider a tank that has been purchased to hold the wine produced in one day but is unfortunately not up to the task. The buyer laments:

(84) The tank is more shallow and narrow than it should to be.

The tank's depth would be ok, if only it were wider. Its width would be ok, if only it were deeper. So one might be reluctant to say that it's shallower than it should be or that it's narrower than it should be. Conjunction at the AP level with *more* attaching above that and then raising produces a predicate that names a shallow-narrow degree.

In the examples considered up to now, *–er* quantifier-raised coming to rest not far from the gradable predicate to which it is attached. But, as Heim (2001) showed, there are modals above which *–er* can raise and, depending on the choice of the modal, one

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[20] Williams (1977:132-3) took *than* to indicate the greater-than relation and he posited occurrences of *more* in both the main and subordinate clauses that quantifier-raise to create amount denoting expressions. von Stechow (1984a:8) thought that couldn't be right given that in his grandmother's German *wie* is used for the standard marker in both the comparative and the equative. He concluded that the greater-than relation must be encoded in the comparative marker. One could raise the same objection based on the use of English *than* in comparatives of inferiority and comparatives of superiority. However, if *–er* and *than* are related by agreement, as Alrenga & Kennedy (2014:43) suggest, the objection loses its force. Comparatives of inferiority might have a *than* that agrees with *less* and has '↘' in its meaning instead of '↗'.

[21] I have in mind something similar to Buccola & Spector (2016:165)'s pragmatic economy constraint, which says:

> An LF φ containing a numeral n is infelicitous if, for some m distinct from n, φ is truth-conditionally equivalent to the result of substituting m for n in φ.

will get 'maximum' or 'minimum' readings. In the example below, *–er* moves above the modal verb *had to*:

(85) a. The wire was longer than it had to be.
b. than OP λ_2 [$\exists\sigma$ [*–er* λ_1 it had to be t_1 long] t_2]
c. $[\![\lambda_1\ \textit{it had to be } t_1 \textit{ long}]\!]^{w,g,c} = \lambda\langle x,w'\rangle$[it had to be $\langle x,w'\rangle$ long]

(85)c gives the meaning for 'λ_1 it had to be t_1 long', the scope of *–er* in (85)b. (85)c has a degree meaning, type $\langle e{\times}s, t\rangle$. It is true of possible individuals that the wire has to meet or exceed in length. If the wire is required to be at least 2 inches long, then (85)c will be true of any actual or possible individual 2 inches long or less. That set of possible individuals just is the degree of length that a 2-inch long individual has. In that case, (85)a says that the wire is more than 2 inches long. What (85) expresses is that the wire exceeds its minimum required length. Replacing *had to* with *allowed to,* we get:

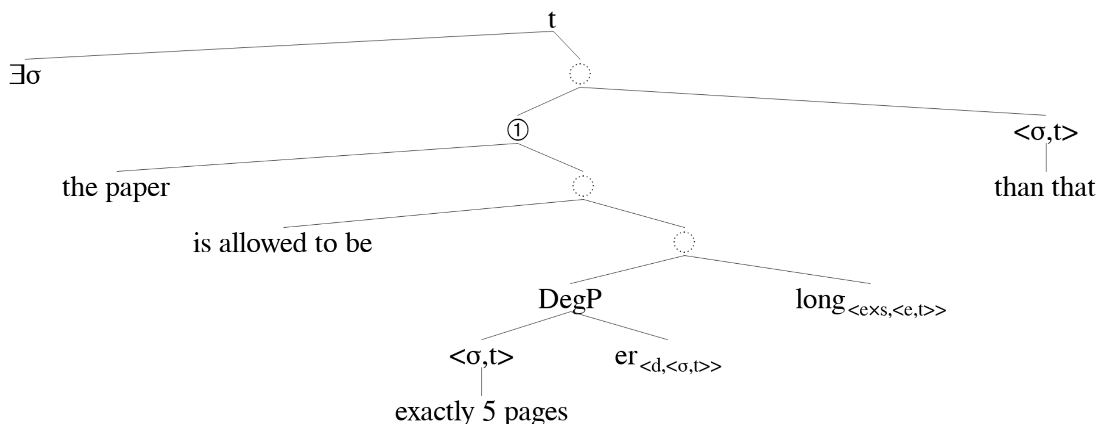
(86) a. The wire was longer than it was allowed to be.
b. than OP λ_2 [$\exists\sigma$ [*–er* λ_1 it was allowed to be t_1 long] t_2]
c. $[\![\lambda_1\ \textit{it was allowed to be } t_1 \textit{ long}]\!]^{w,g,c} = \lambda\langle x,w'\rangle$[it was allowed to be $\langle x,w'\rangle$ long]

(86)c gives the meaning for 'λ_1 it was allowed to be t_1 long', the scope of *–er* in (86)b. (86)c has a degree meaning, type $\langle e{\times}s, t\rangle$. It is true of possible individuals that the wire is allowed to meet or exceed in length. If the wire was allowed to be up to 4 inches long, then (86)c will be true of any actual or possible individual 4 inches long or less. That set of individuals just is the degree of length that a 4-inch long individual has. So (86)a says that the wire was more than 4 inches long. What (86) expresses is that the wire exceeded its maximum allowable length.

Having seen that *–er* scopes over modals in the comparative clause, we now expect to find *–er* scoping over modals in the main clause of a comparative. In fact, it was this context in which Heim (2001) first identified the effects of scoping above necessity and possibility modals. (87) below is one of her examples. Although its analysis has been questioned (Oda 2008 cited in Beck 2012:259ff), it is instructive to see how it works in the current context.

(87) (This draft is 10 pages.) The paper is allowed to be exactly 5 pages longer than that.
(88) "it is exactly 15pp long in the acceptable worlds where it is longest, which means it is not allowed to be longer than 15pp." (Heim 2001)

(87) has a kind of comparative different from the ones we've seen so far. The object of *than* is a pronoun referring to a previously mentioned degree. It requires a meaning for *than* as in (89). To produce the reading in (88), *than* would occur above *allowed* and *–er* would create a segmental predicate by moving to adjoin to the node labeled ①.



(89) $[\![\textit{than}]\!]^{w,g,c} = \lambda d\, \lambda\sigma\, \nearrow(\sigma) \wedge \textsc{start}(\sigma)=d$

In this case, it's imperative that the measure phrase be generated inside DegP, otherwise it will be left behind when *–er* raises and will be interpreted in a non-segmental environment. So if Heim's analysis is correct, we have an argument for that syntax.

As observed in (86)-(88), when *–er* raises above an existential, it gives rise to a maximal reading. We can see this in the individual domain using negative polarity items. As long as there is no *exactly* differential in the main clause (Rullmann 1995:106), the scope of *–er* in the comparative clause is a downward entailing environment[22]. When a negative polarity existential is found there, it gives rise to a maximal reading. (90) says the box's weight exceeds the weight of the heaviest other object, unlike (91) which says only that it exceeded the weight of one of them.

(90) The box was heavier than any of the tubes were
(91) The box was heavier than one of the tubes was.

An important feature of the proposal made here has to do with the notion of "scope of comparison" (Williams 1974:217-218, Gawron 1995, Bhatt & Pancheva 2004). Compare the following:

(92) a. Jack wanted the coat to be more expensive than Jill did.
b. Jack wanted [the coat to be more expensive than the sweater was].

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[22] *–er* combines with a degree denoting expression, which is to say an expression that denotes a set of possible individuals. According to the semantics described above, *Jack is taller than Jill is* is true if Jill's height is a proper subset of Jack's. If we replace the expression denoting Jill's height with one that denotes a subset, then surely that smaller set will be also be proper subset of Jack's height. In other words, if the comparative statement $\varphi(A)$ is true, where A is the scope of *–er* in the comparative clause, and $[\![B]\!]^{w,g,c} \subseteq [\![A]\!]^{w,g,c}$, then $\varphi(B)$ is true.

I'm using *any* for illustration even though there is some question about its NPI status in this context (Aloni & Roelofsen 2014). There are uncontroversial cases of NPIs, verbs and adverbs, that appear in comparative clauses.

In (92)a, *want* is included in the scope of comparison. We compare <u>prices that Jack wanted</u> to the ones that Jill wanted. In (92)b, we report on Jack's desires in which the <u>price of the coat</u> is compared to that of the sweater. In (92)b, the scope of comparison does not extend past the clause embedded under *want.* Our logical forms have two mobile operators in them, OP and *–er*. OP defines the scope of comparison, while *–er* defines what we might call the scope of degree. In (79), *Jack expects the engine to be one boxcar longer than Jill does*, the verb *expect* and the differential (*one boxcar*) are in the scope of comparison but not in the scope of degree. This presents a challenge to previous analyses for various reasons. For some, there is just one operator moving in the clause, which means that scope of comparison and scope of degree are conflated and that gives the wrong interpretation for *expect.* For some, there are two operators, but they differ between the main clause and the comparative clause. That means that to provide the right input for ellipsis, the scope-of-degree operator needs to be raised above *expect* (Alrenga & Kennedy 2014).

Summarizing now, the comparative marker, *–er,* turns out to be little more than a type shifter taking predicates of possible individuals into predicates of segments. Its presence is needed to form the input to the standard marker *than* in which the crux of comparison resides. In order to do its job, *–er* needs to move to take scope. In so doing, it divides the clause, giving rise to an interesting interaction of degree and quantification.[23] NPI quantifiers and some modals remain below *–er*, other modals and non-modal quantifiers raise above the segment-existential that binds the scalar segment argument introduced by *–er*.

6 Conclusion

Presumably we learn the meaning of *cold* by associating it with the physical sensation of coldness. Maybe at first, that's all there is to it. But eventually, to grasp its meaning we need to appeal to our ability to discriminate among those sensations and the objects that produce them to a greater or lesser degree. That's the message of our first thesis. *cold* is relational. According to our second thesis, when we communicate a comparative judgment we do so by reference to a scalar segment, which is a path of sorts connecting two points. It is natural then that we draw, by analogy, on vocabulary associated with movement in space or time[24]. Simultaneous adoption of these twin theses

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[23] Unlike Heim (2006), the proposal here gives the wrong results when a DP of the form '*exactly* n NP' occurs in the comparative clause. In some ways the present proposal differs from Heim (2006) in the way that Gajewski (2009)'s E-theory differed from Schwarzschild & Wilkinson (2001) and those two theories also differed on the coverage of *exactly* DPs. Gajewski appealed to an implicature-generating mechanism taking scope over the comparative. Zhang (this volume) adopts this same strategy making use of developments in Bumford (2017) and Brasoveanu (2013) to separate out the upper bounding part of *exactly*'s content.

[24] According to Stassen (2006), "the type(s) of comparative construction that a language may employ is argued to be limited by the options that the language has in the encoding of (simultaneous or consecutive) sequences of events."

has led to a new perspective on the interaction of degrees and quantifiers, a rudimentary typology, and a clarification of the use of measurement jargon and of degree structures. In working out the consequences of adopting these hypotheses, I've only covered a small part of what is known about gradable predicates and degree constructions. There are challenges awaiting in the various types of adjectives and many kinds of degree constructions left to be considered, including kinds of comparatives.

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